

MA 345-12 Differential Equations

Text : “*Fundamentals of Differential Equations*”
by Nagle, Saff and Snider

Instructor : Dr. E. Jacobs

Office : COAS 301-36

Office Hours:

Mon, Wed, Fri.: 2:00 - 2:30 and 4:15 - 4:45 PM
and Tues: 2:00 - 3:00 PM

e-mail: jacobs50@xecu.net

MA 345 Class meeting times

MWF 5:00 - 5:50 in COAS 305

Tues 5:15 - 6:30 in COAS 402

Grading System

Exam Average 85%

Homework Average 15%

There will be four exams, all equally weighted.

Exam 4 will be the final exam.

Overall Average	Grade in Course
-----------------	-----------------

90 - 100	A
----------	----------

80 - 89	B
---------	----------

70 - 79	C
---------	----------

60 - 69	D
---------	----------

0 - 59	F
--------	----------

Some important dates:

Exam 1 Tuesday, January 28

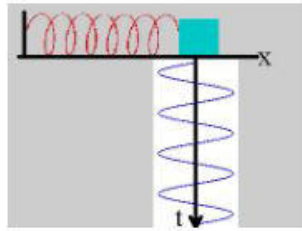
Assignment 1 Monday, January 13

Rules for Exams

1. You may not have formulas or notes with you on exams.
2. Put away cell phones and smart watches during exams.
3. Make-up exams will only be given for in very special circumstances. Arrangements for a make-up exam must be made within 24 hours of the original exam.

Homework

1. Homework must be neat. Show work.
2. Upload to Canvas in .pdf format
3. Each assignment should be submitted as ONE file
4. Homework must be handed in on time.



- [Course Syllabus](#) ↓
- [Welcome to Differential Equations](#)
- [Complete List of Homework Assignments](#) ↓
- [How to upload your homework](#) ↓
- [Office Hours This Week](#)

[Click here to go to Module 1 page](#)

Modules:

- [Module 1 - First Order Equations](#)
- [Module 2 - Matrix Methods and Homogeneous Linear Differential Equations](#)
- [Module 3 - Nonhomogeneous Linear Differential Equations and Laplace Transforms](#)

Module 1 - First Order Equations



In this module, you will learn methods to solve equations of the form:

$$\frac{dy}{dx} = f(x, y)$$

Here are the assignments for this module:

[Assignment 1.](#)

[Assignment 2](#)

[Assignment 3](#)

Assignment 1. *Introduction to Differential Equations. Separation of Variables*

Read 1.1, 1.2, 2.1 and 2.2

You should be able to do the following problems:

Exercise 1.2 Problems 1 - 15, Exercise 2.2 Problems 7 - 26

Hand in the following problems:

Solve the following differential equations for problems 1 - 3.

1.
$$\frac{dy}{dx} = \frac{y}{2\sqrt{x}} \quad y(1) = 1$$

2.
$$\frac{dy}{dx} = \frac{y(y+1)}{x}$$

3.
$$\frac{dy}{dx} + xy = x \quad y(0) = 2$$

4. Scientists at the University of Nebraska Medical Center performed an experiment to determine the rate at which pancreatic cancer cells grow. Approximately 500,000 pancreatic cancer cells were injected into the pancreas of each laboratory rat and the number of cells was observed growing over a three week period. If $y(t)$ represents the population of cancer cells in a particular rat after t hours, then $y(t)$ solves the following differential equation:

$$\frac{dy}{dt} = ky \quad (\text{where } k \text{ is a positive constant})$$

a. Solve this differential equation and obtain a formula for $y(t)$.

b. The scientists determined experimentally that $k = 0.05 \ln 2$. Calculate how long it takes for the population of cancer cells to double.

5. Let $M(t)$ denote the mass of a radioactive object after t years. The fact that the rate at which the mass is decreasing is proportional to the mass itself leads to the differential equation:

$$\frac{dM}{dt} = -\lambda M \quad \text{where } \lambda \text{ is a constant}$$

a. Solve this differential equation and obtain a formula for $M(t)$.

b. Suppose for a particular radioactive substance we begin with 4 grams. If the mass is down to 1 gram after 1 year, calculate the constant λ

Attendance

Attendance is not counted in your grade in the course except for borderline cases.

Differential Equation

A differential equation is an equation that involves the derivative of an unknown function.

$$\frac{dy}{dx} - 2y = 0$$

$$\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} + y = e^x$$

A differential equation could involve higher order derivatives

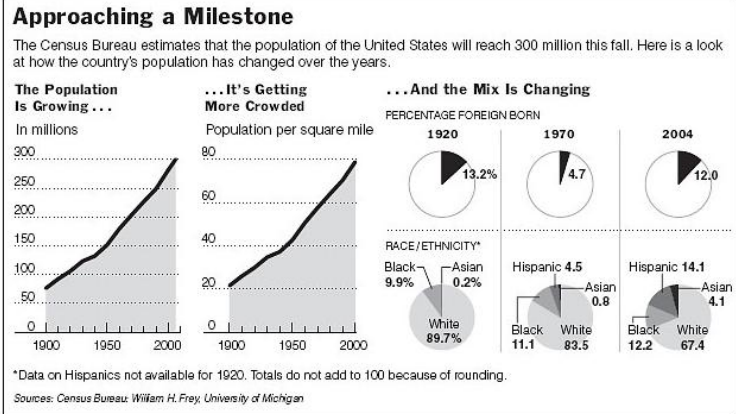
$$m \frac{d^2 y}{dt^2} + \beta \frac{dy}{dt} + ky = 0$$

A differential equation could involve partial derivatives

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad \text{where } u = u(x, t)$$
$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi \quad \text{where } \Psi = \Psi(x, t)$$

Let $P = P(t)$ be the population at time t .

January 13, 2006



By The New York Times

Malthusian Model of Growth

Population grows at a rate proportional to the size of the population at any point in time.

Malthusian Model of Growth

Population grows at a rate proportional to the size of the population at any point in time.

$\frac{dP}{dt}$ is proportional to P at any t

Malthusian Model of Growth

Population grows at a rate proportional to the size of the population at any point in time.

$\frac{dP}{dt}$ is proportional to P at any t

$$\frac{dP}{dt} = kP$$

Let $A = A(t)$ be the amount of money in a bank account after t years. The bank pays 1 percent interest. Find a formula for $A(t)$.

Let $A = A(t)$ be the amount of money in a bank account after t years. The bank pays 1 percent interest. Find a formula for $A(t)$.

$$\text{Interest} = (\text{Principal})(\text{rate})(\text{time})$$

Let $A = A(t)$ be the amount of money in a bank account after t years. The bank pays 1 percent interest. Find a formula for $A(t)$.

$$\text{Interest} = (\text{Principal})(\text{rate})(\text{time})$$

$$dA = A \cdot 0.01 \cdot dt$$

Let $A = A(t)$ be the amount of money in a bank account after t years. The bank pays 1 percent interest. Find a formula for $A(t)$.

$$\text{Interest} = (\text{Principal})(\text{rate})(\text{time})$$

$$dA = A \cdot (0.01) \cdot dt$$

$$\frac{dA}{dt} = 0.01A$$

Population problem:

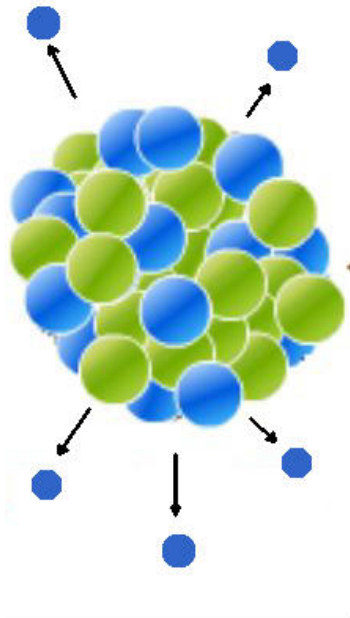
$$\frac{dP}{dt} = kP$$

Bank account problem:

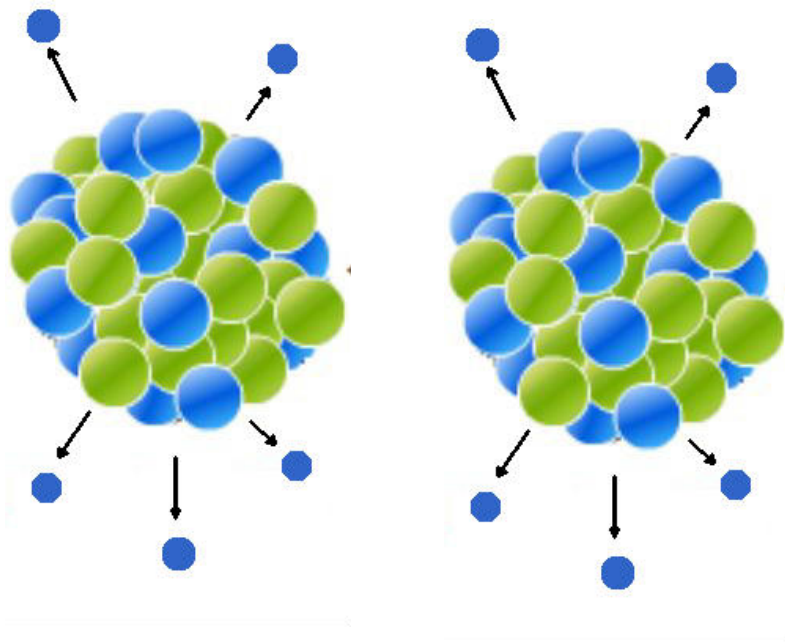
$$\frac{dA}{dt} = 0.01A$$

The mass of a radioactive object decreases with time:

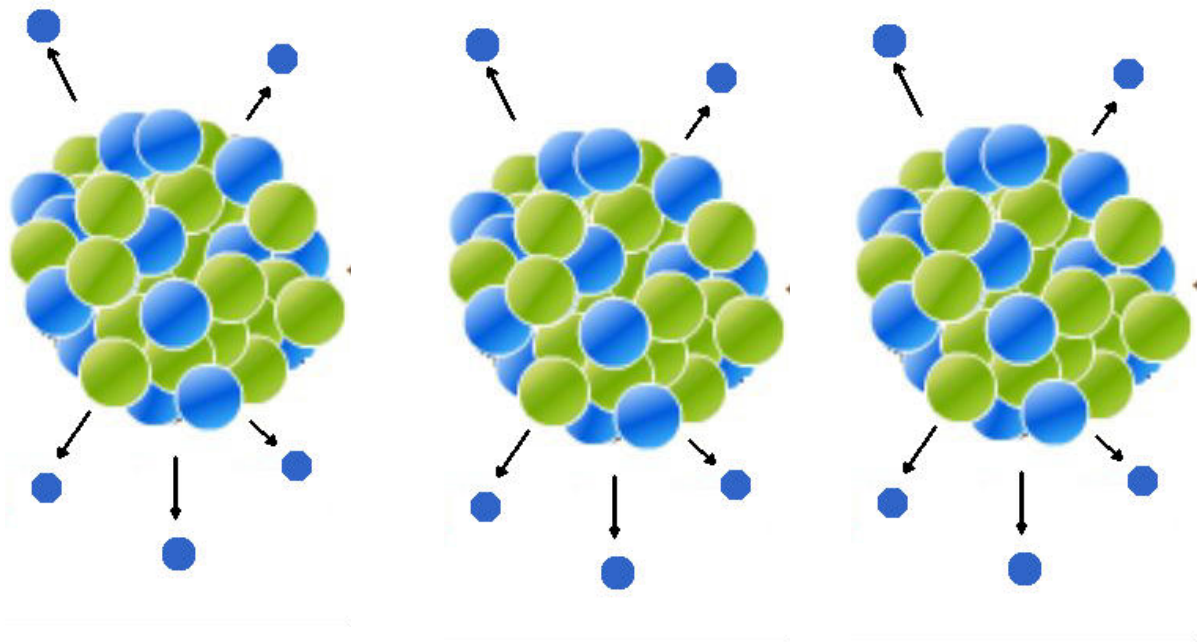
$$M = M(t)$$



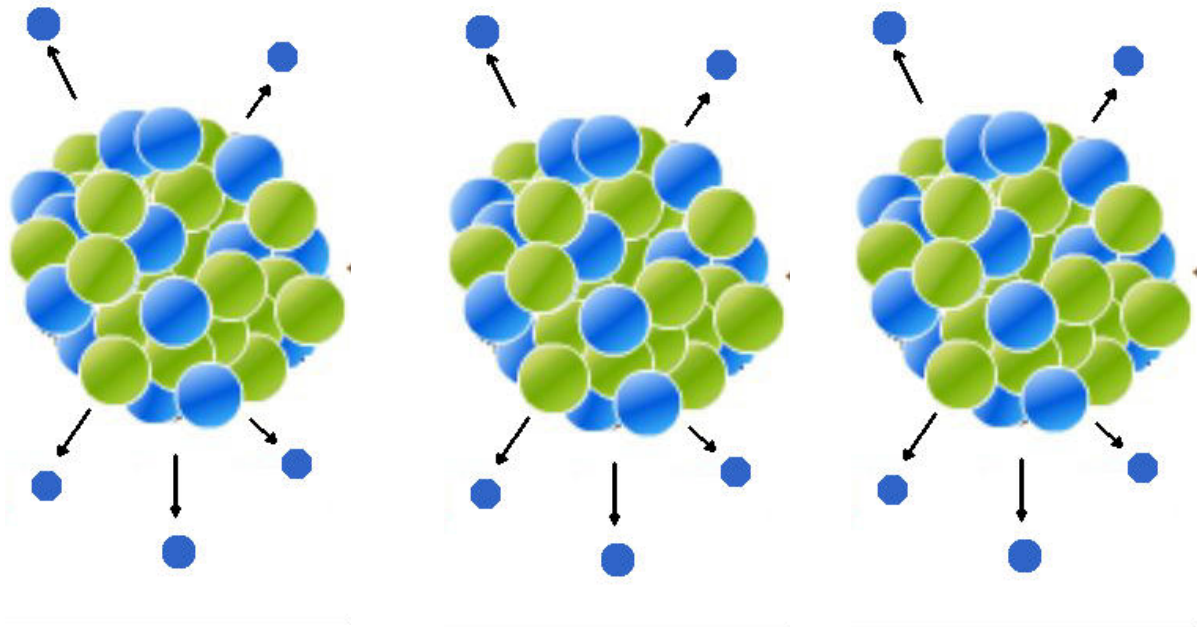
If we double the mass, we double the rate at which the mass decreases.



If we triple the mass, we triple the rate at which the mass decreases.

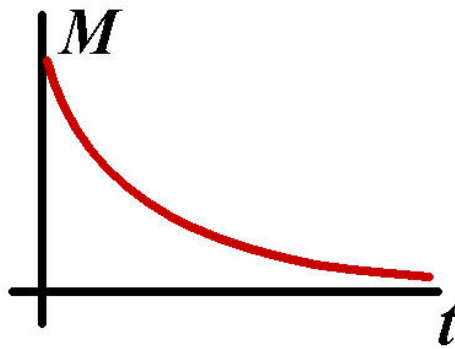


The rate at which the mass decreases is proportional to the mass at any given time.



The rate at which the mass decreases is proportional to the mass at any given time.

$$\frac{dM}{dt} = -\lambda M$$

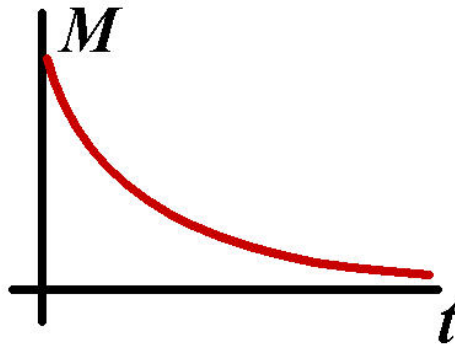


The rate at which the mass decreases is proportional to the mass at any given time.

$$\frac{dM}{dt} = \lambda M \quad (\lambda \text{ is a negative number})$$

Let $M = M(t)$ be the mass of a radioactive object

$$\frac{dM}{dt} = -\lambda M$$



Population Growth

$$\frac{dP}{dt} = kP$$

Bank Account Problem

$$\frac{dA}{dt} = 0.01A$$

Radioactive Mass

$$\frac{dM}{dt} = -\lambda M$$

$$\frac{dP}{dt} = kP \quad \frac{dA}{dt} = 0.01A \quad \frac{dM}{dt} = -\lambda M$$

Find a formula for $y = y(x)$

$$\frac{dy}{dx} = ky$$

Spring Problem:

$$m \frac{d^2 y}{dt^2} + \beta \frac{dy}{dt} + ky = 0$$

Electrical Circuit Problem:

$$L \frac{d^2 Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = 0$$

How do we solve a differential equation?

$$\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -2x$$

$$y = \int -2x \, dx = -x^2 + C$$

$$\frac{dy}{dx} = -2x$$

$$dy = -2x dx$$

Separation of Variables

$$\frac{dy}{dx} = -2x$$

$$dy = -2x dx$$

$$\int dy = \int -2x dx$$

$$y + C_1 = -x^2 + C_2$$

$$\frac{dy}{dx} = -2x$$

$$dy = -2x dx$$

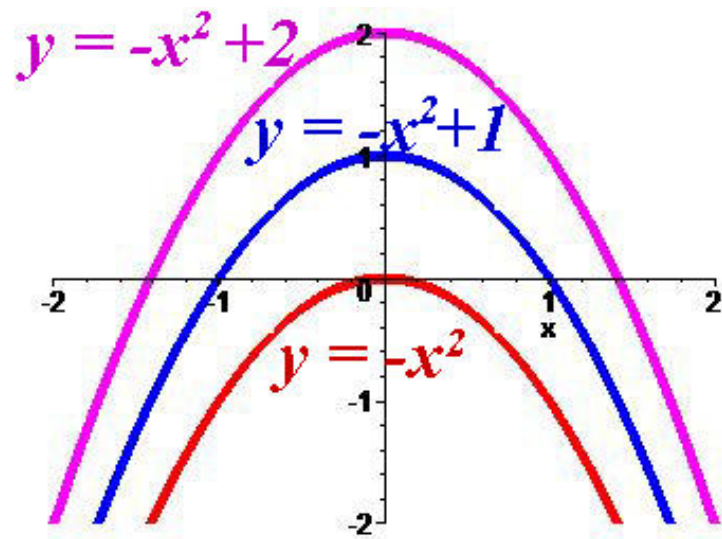
$$\int dy = \int -2x dx$$

$$y + C_1 = -x^2 + C_2$$

$$y = -x^2 + C$$

(where $C = C_2 - C_1$)

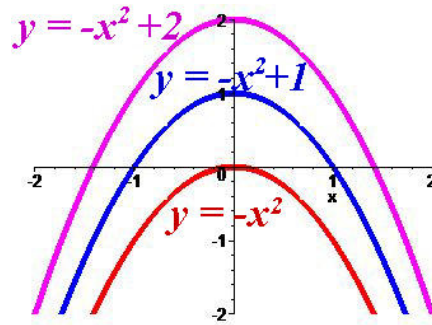
$$y = -x^2 + C$$



$$\frac{dy}{dx} = -2x$$

$$y = -x^2 + C$$

This is called the *general solution*

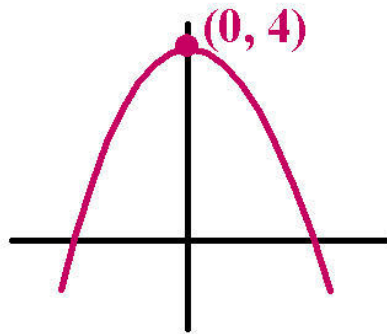


$$\frac{dy}{dx} = -2x \quad \text{where } y(0) = 4$$

$$y = -x^2 + C$$

If $x = 0$ then $4 = -0^2 + C = C$

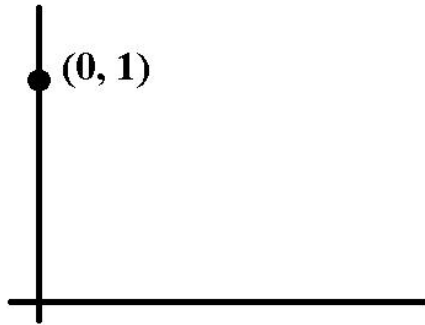
$$y = -x^2 + 4 \quad (\text{Particular Solution})$$



$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

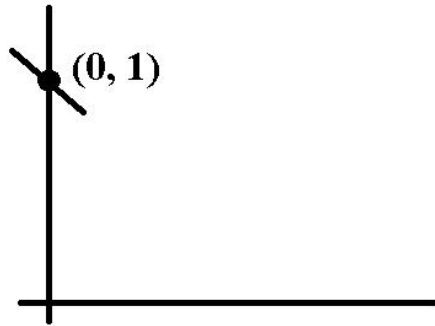
$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

The curve passes through $(0, 1)$.



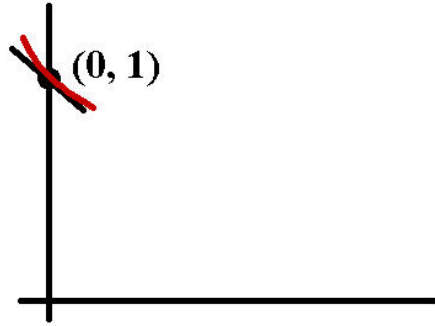
$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

The slope is -2 at $x = 0$



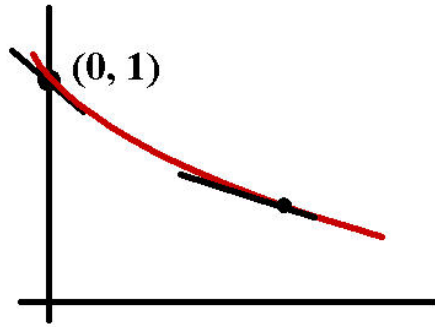
$$\frac{dy}{dx} = -2y \quad \frac{d^2y}{dx^2} = -2\frac{dy}{dx} = 4y > 0$$

The curve is concave up at $x = 0$



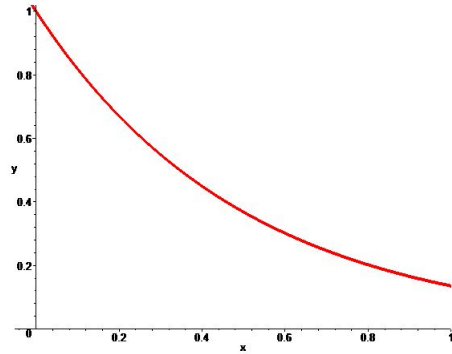
$$\frac{dy}{dx} = -2y$$

As we move to the right, y is getting close to 0 so the slope is getting closer to 0.



$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

The shape of the curve is becoming more evident.



$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

Integrate both sides.

$$y = \int -2y \, dx$$

$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

Integrate both sides.

$$y = \int -2y \, dx = ???$$

$$\int -2 \sin x \, dx = 2 \cos x + C$$

$$\int -2e^x \, dx = -2e^x + C$$

$$\int -2x^3 \, dx = -\frac{1}{2}x^4 + C$$

$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

$$dy = -2y dx$$

$$\frac{1}{y} dy = -2 dx$$

Separation of variables

$$\frac{dy}{dx} = -2y \quad \text{where } y(0) = 1$$

$$dy = -2y dx$$

$$\frac{1}{y} dy = -2 dx$$

$$\int \frac{1}{y} dy = \int -2 dx$$

$$\ln |y| = -2x + C$$

$$\ln |y| = -2x + C$$

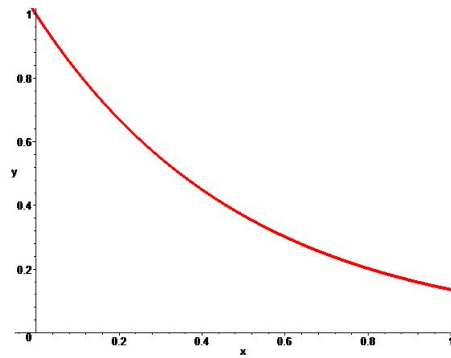
$$|y| = e^{-2x+C}$$

$$y = \pm e^{-2x+C} = (\pm e^C) e^{-2x} = ae^{-2x}$$

$$y = ae^{-2x}$$

Since $y(0) = 1$, we get $1 = ae^0 = a$

$$y = 1e^{-2x}$$



$$\frac{dy}{dx} = 2xy - 2xy^2$$

Find the general solution

$$\frac{dy}{dx} = 2xy - 2xy^2 = 2xy(1 - y)$$

$$\frac{1}{y(1 - y)} dy = 2x dx$$

$$\int \frac{1}{y(1 - y)} dy = \int 2x dx$$

$$\frac{dy}{dx} = 2xy - 2xy^2 = 2xy(1 - y)$$

$$\frac{1}{y(1 - y)} dy = 2x dx$$

$$\int \frac{1}{y(1 - y)} dy = \int 2x dx$$

$$\int \left(\frac{1}{y} + \frac{1}{1 - y} \right) dy = \int 2x dx$$

$$\ln |y| - \ln |1 - y| = x^2 + C$$

$$\ln |y| - \ln |1 - y| = x^2 + C$$

$$\ln \left| \frac{y}{1 - y} \right| = x^2 + C$$

$$\left| \frac{y}{1 - y} \right| = e^{x^2 + C} = e^C e^{x^2}$$

$$\frac{y}{1 - y} = \pm e^C e^{x^2}$$

$$\frac{y}{1 - y} = a e^{x^2}$$

$$\frac{y}{1-y} = ae^{x^2}$$

$$y = ae^{x^2} (1-y)$$

$$y = ae^{x^2} - ae^{x^2} y$$

$$y + ae^{x^2} y = ae^{x^2}$$

$$(1 + ae^{x^2}) y = ae^{x^2}$$

$$y = \frac{ae^{x^2}}{1 + ae^{x^2}}$$

$$\frac{dy}{dx} = x - 2y$$

Solve.

$$\frac{dy}{dx} = x - 2y$$

Solve. Try separation of variables.

$$dy = (x - 2y)dx$$

$$dy - x dx = -2y dx$$

$$dy + 2y dx = x dx$$

$$\frac{1}{x - 2y} dy = dx$$

Now what?