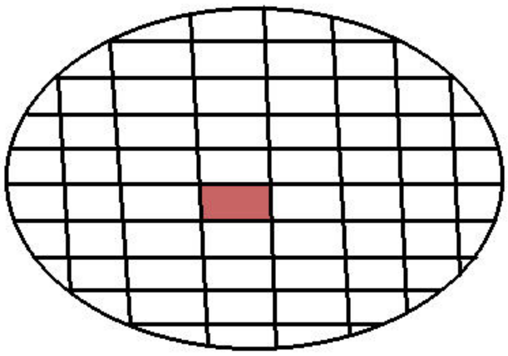
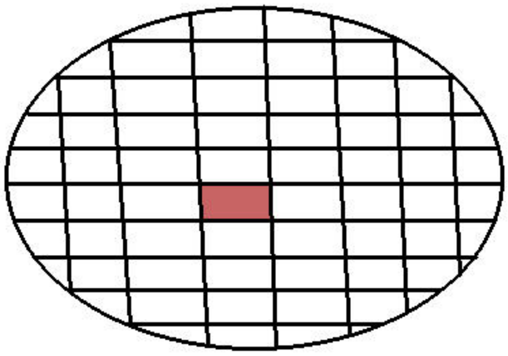
$$\vec{\mathbf{F}} = \langle F_1(x, y),\ F_2(x, y)\rangle$$

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \quad \text{is the circulation per unit area at a point}$$

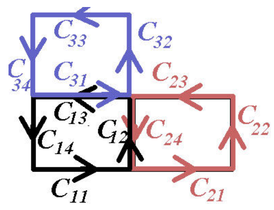
Subdivide a region R into a grid.
On any grid section, $\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\, dA$ is the circulation around this section.

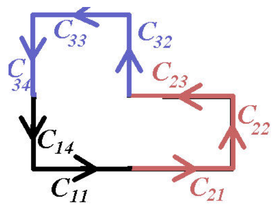


$$\int\int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA \text{ is the sum of all of these circulations}$$

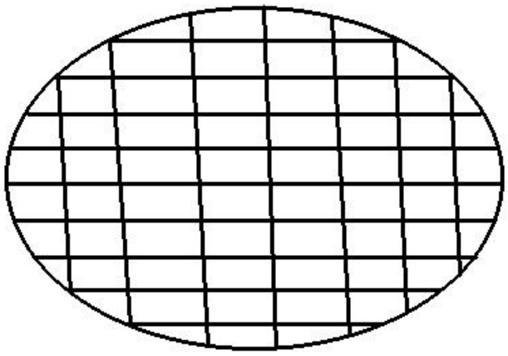


The line integrals over interior boundaries cancel.

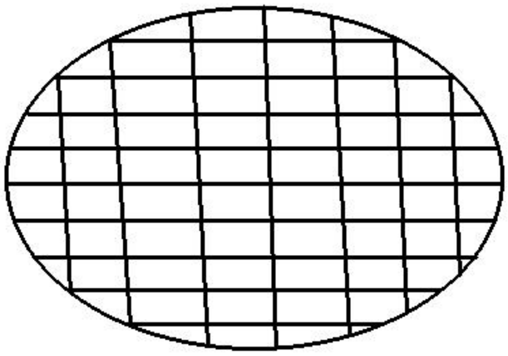




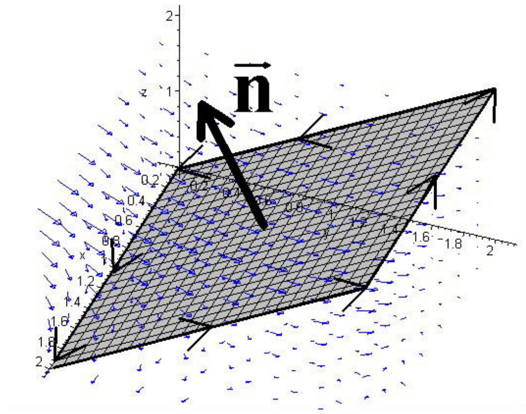
Thus $\int\int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\, dA$ adds up to the line integral around the outer boundary C.



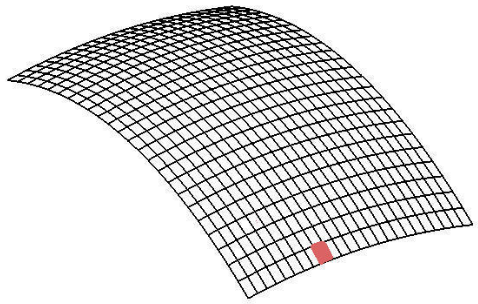
$$\int\int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\, dA \;= \oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$$



In higher dimension, $(\nabla \times \vec{\mathbf{F}}) \bullet \vec{\mathbf{n}}$ is the circulation per unit area in a plane perpendicular to $\vec{\mathbf{n}}$.

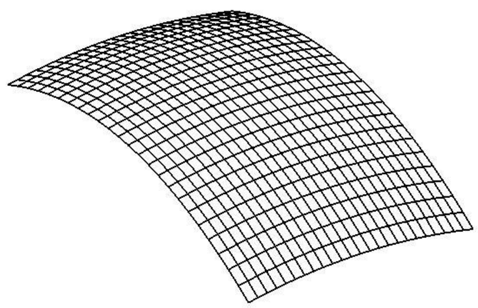


Divide a surface S into a grid. $(\nabla \times \vec{\mathbf{F}}) \bullet \vec{\mathbf{n}}\, dS$ is the circulation around one section.

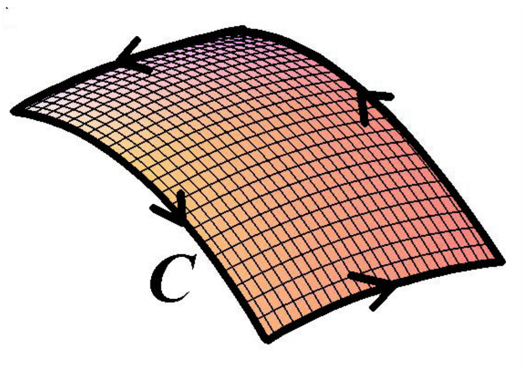


Add up the circulations over all these sections.

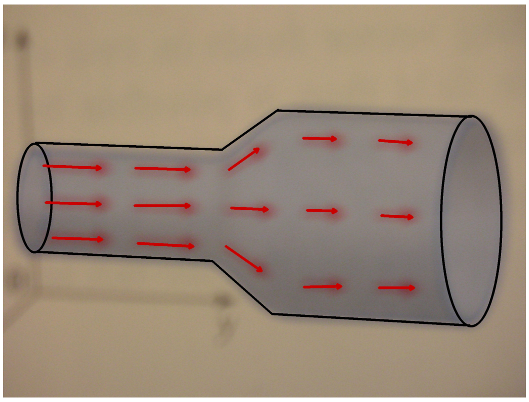
$$\int\int_S (\nabla \times \vec{\mathbf{F}}) \bullet \vec{\mathbf{n}}\, dS$$



$$\int\int_S (\nabla \times \vec{\mathbf{F}}) \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$$

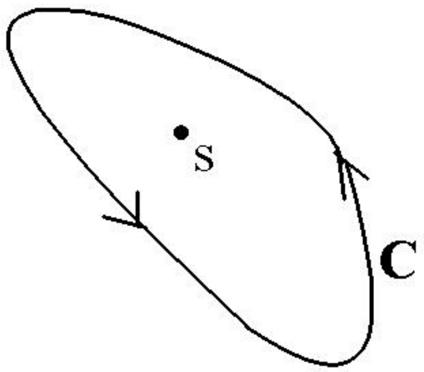


Let $\vec{\mathbf{v}}$ be the velocity vector field of a fluid.

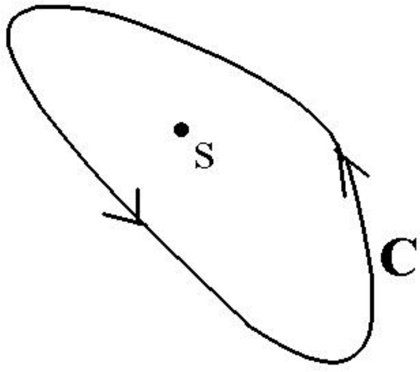


For an *ideal fluid*, the circulation is always zero.

$$\oint_C \vec{\mathbf{v}} \bullet d\vec{\mathbf{r}} = 0$$

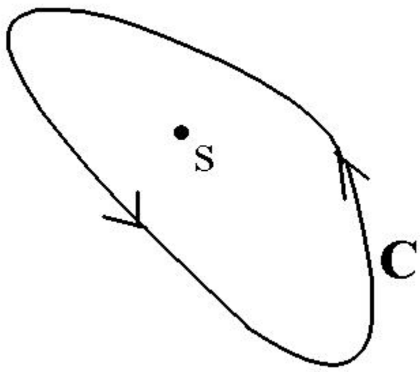


$$\int\int_S (\nabla \times \vec{\mathbf{v}}) \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{v}} \bullet d\vec{\mathbf{r}} = 0$$



If $\iint_S (\nabla \times \vec{\mathbf{v}}) \bullet \vec{\mathbf{n}}\, dS = 0$ for every surface S then we may conclude that:

$$\nabla \times \vec{\mathbf{v}} = \vec{\mathbf{0}} \text{ at all points}$$



There are two equivalent ways of describing ideal fluids:

$$\oint_C \vec{\mathbf{v}} \bullet d\vec{\mathbf{r}} = 0 \text{ for every closed loop}$$

$$\nabla \times \vec{\mathbf{v}} = \vec{\mathbf{0}} \text{ at every point}$$