Heat equation in one dimension: $u = u(x, t)$

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

Heat equation in two dimensions: $u = u(x, y, t)$

$$\frac{\partial u}{\partial t} = \alpha^2 \nabla^2 u$$

Heat equation in two dimensions: $u = u(x, y, t)$

$$\frac{\partial u}{\partial t} = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

When the temperature has reached a *steady-state*, $u = u(x, y)$

$$\frac{\partial u}{\partial t} = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

reduces to:

$$0 = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Laplace's Equation

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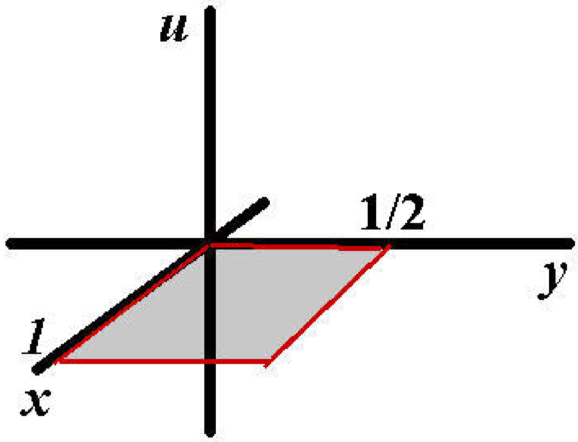
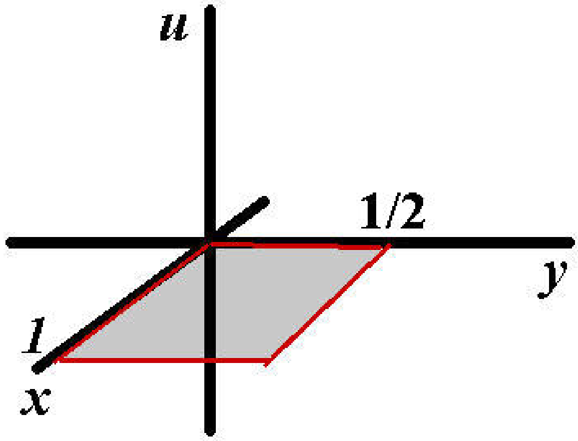
Laplace's Equation



$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$u(0, y) = 0 \quad u\left(x,\ \tfrac{1}{2}\right) = 0 \quad u(x, 0) = 0 \quad u(1, y) = 1$$



$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Begin by looking for all nontrivial solutions of the form:

$$X(x)Y(y)$$

$$u(x,y)=\sum_{n=1}^{\infty} b_n \sinh(2\pi n x)\sin(2\pi n y)$$

$$\text{where} \qquad b_n=\frac{2}{\pi n \sinh(2\pi n)}\left(1-(-1)^n\right)$$