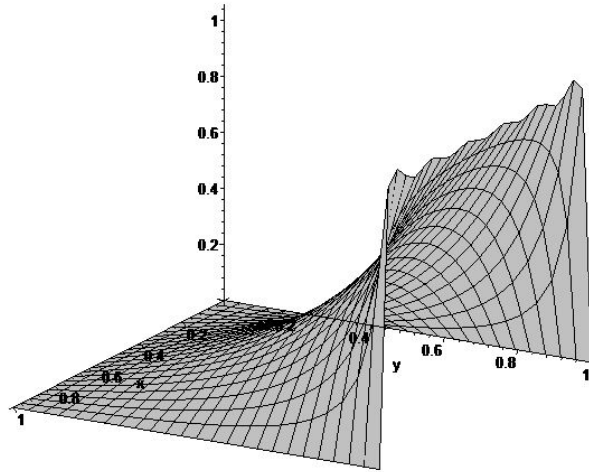Solution of $\nabla^2 u = 0$ with
$u(x, 1) = 1,\ u(x, 0) = 0,\ u(0, y) = 0,\ u(1, y) = 0$

$$u(x, y) = \sum_{n=1}^{\infty} \frac{2(1 - (-1)^n)}{n\pi \sinh n\pi} \sinh n\pi y \sin n\pi x$$
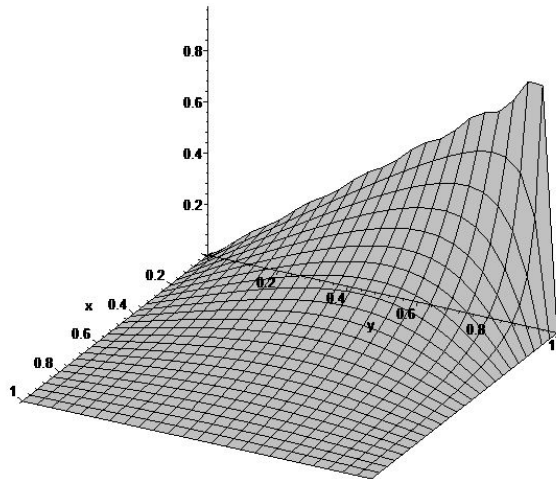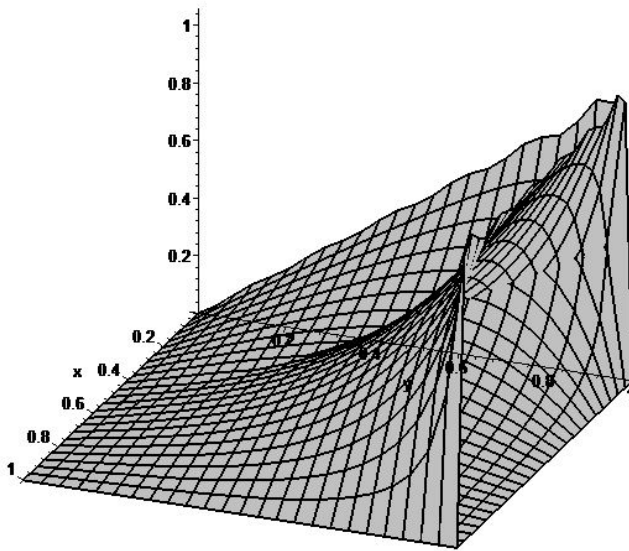
Solution of $\nabla^2 u = 0$ with
$u(x,1) = 0,\ u(x,0) = 0,\ u(0,y) = y,\ u(1,y) = 0$

$$u(x,y) = \sum_{n=1}^{\infty} b_n \left(e^{n\pi x} - e^{2n\pi} e^{-n\pi x}\right) \sin n\pi y$$

where $\quad b_n = \dfrac{-2(-1)^n}{n\pi(1-e^{2n\pi})}$

Solution of $\nabla^2 u = 0$ with
$u(x,1) = 1$, $u(x,0) = 0$, $u(0,y) = y$, $u(1,y) = 0$

Solution of heat equation with $u(0, t) = 1$, $u(1, t) = 3$ and $u(x, 0) = 6x^2 - 4x + 1$

$$u = 2x + 1 + \sum_{n=1}^{\infty} \frac{24\left((-1)^n - 1\right)}{n^3 \pi^3} e^{-(n\pi\alpha)^2 t} \sin n\pi x$$