d'Alembert's Formula

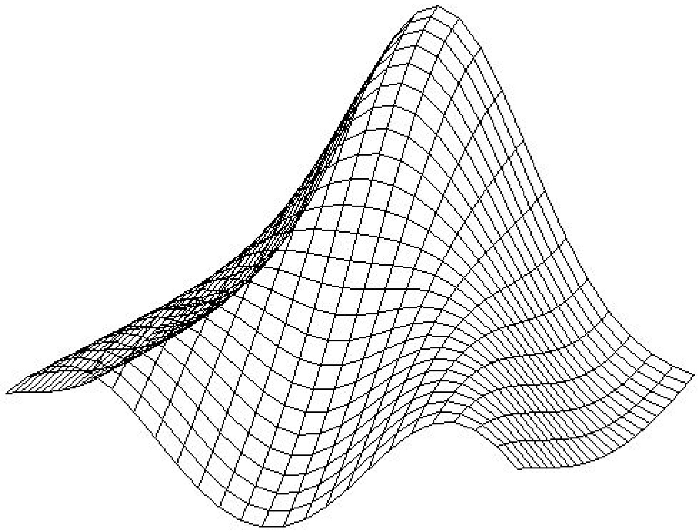
$$u(x,t) = \frac{1}{2}(f(x+ct) + f(x-ct))$$

Example:

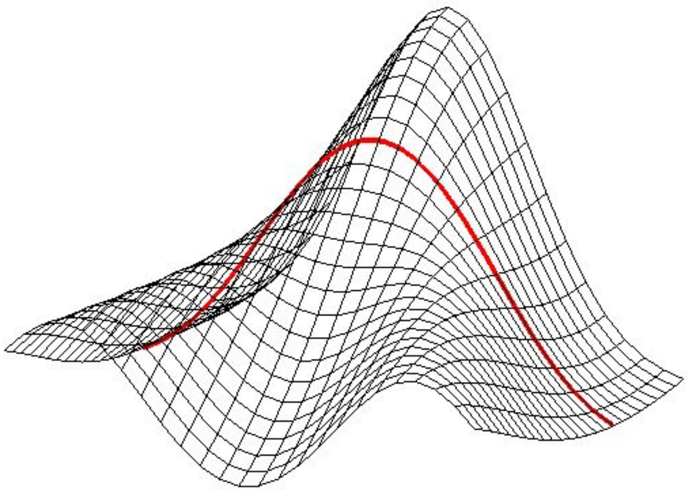
Take $f(x) = e^{-x^2}$

$$u(x,t) = \frac{1}{2}\left(e^{-(x+ct)^2} + e^{-(x-ct)^2}\right)$$

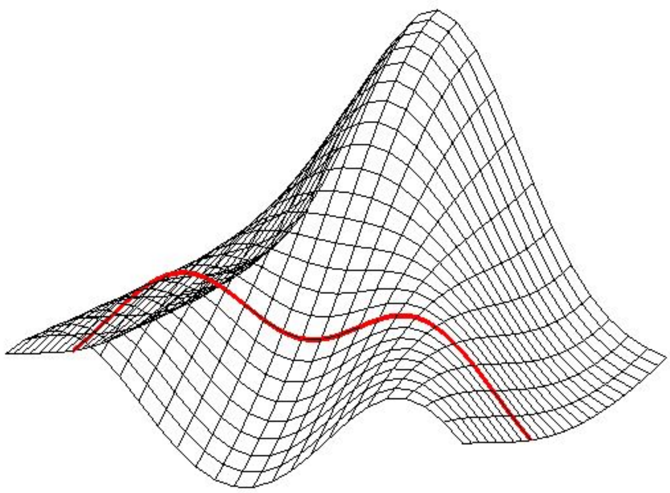
$$u(x,t) = \frac{1}{2}\left(e^{-(x+ct)^2} + e^{-(x-ct)^2}\right)$$



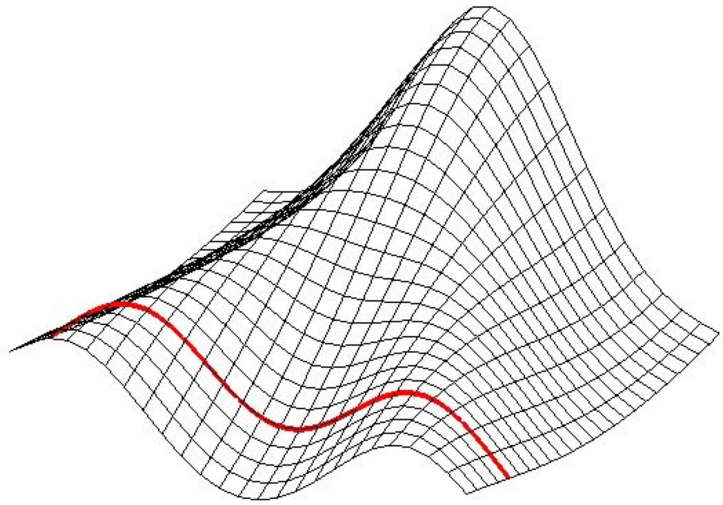
$$u(x,t) = \frac{1}{2}\left(e^{-(x+ct)^2} + e^{-(x-ct)^2}\right)$$



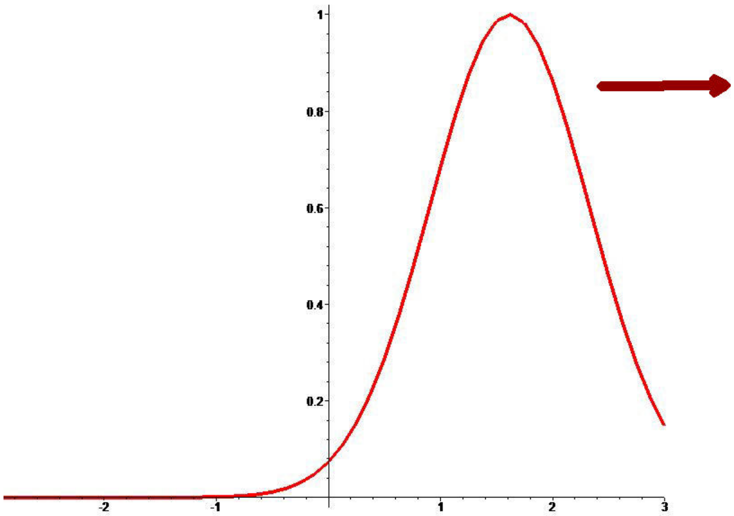
$$u(x,t) = \frac{1}{2}\left(e^{-(x+ct)^2} + e^{-(x-ct)^2}\right)$$



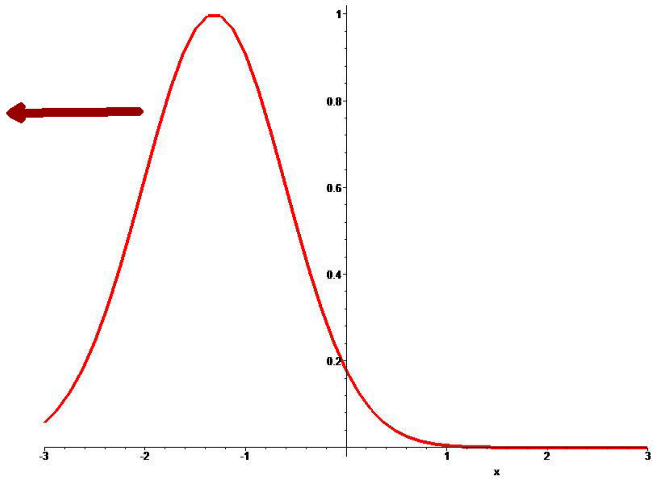
$$u(x,t) = \frac{1}{2}\left(e^{-(x+ct)^2} + e^{-(x-ct)^2}\right)$$



$f(x - ct)$ represents motion of a wave to the right at speed c



$f(x+ct)$ represents motion of a wave to the right at speed c



d'Alembert's Formula

$$u(x,0) = f(x) \qquad \frac{\partial u}{\partial t}(x,0) = 0$$

$$u(x,t) = \frac{1}{2}(f(x+ct) + f(x-ct))$$