$$\int_a^b f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = 0 \text{ for } \textit{every} \text{ interval } [a,\ b]$$

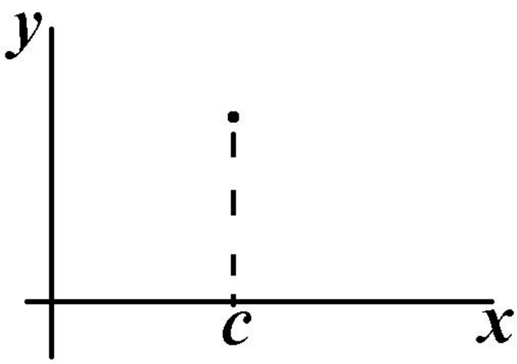
If $f(x) = 0$ for every value of x then

$$\int_a^b f(x)\,dx = 0 \text{ for every interval } [a,\ b]$$

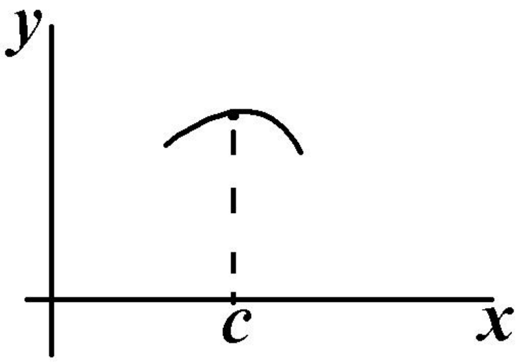
$$\int_a^b f(x)\,dx = 0 \text{ for every interval } [a,\ b]$$

The *only* continuous function for which this is true is the constant function $f(x) = 0$.

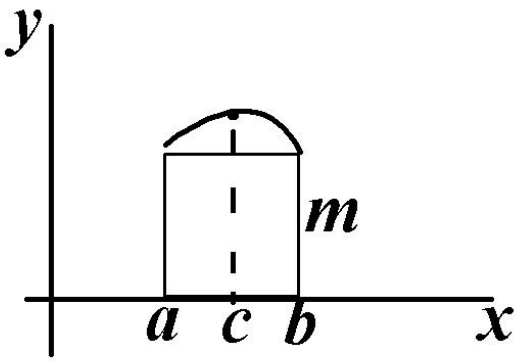
Suppose there were a point c at which $f(c) > 0$



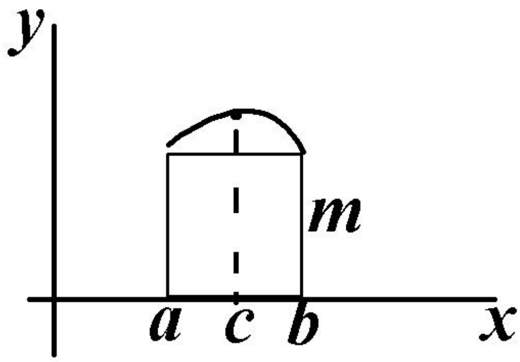
Suppose there were a point c at which $f(c) > 0$. If $f(x)$ is continuous, there will be some interval around c for which $f(x) > 0$.



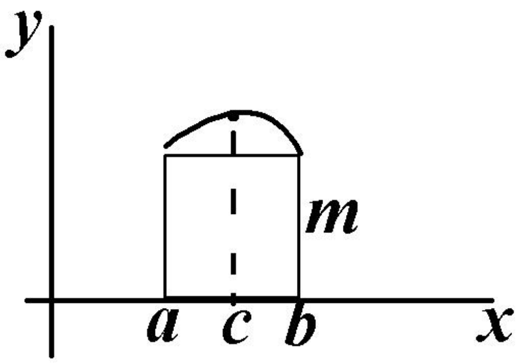
$$\int_a^b f(x)\,dx > m(b-a) > 0$$



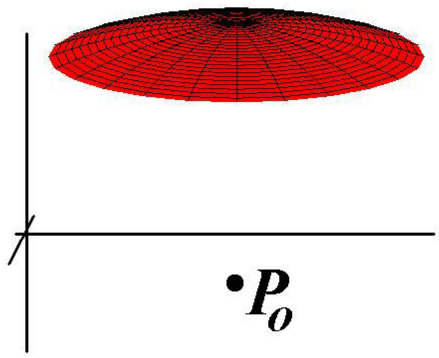
We have found an interval $[a,\ b]$ for which $\int_a^b f(x)\ dx \neq 0$



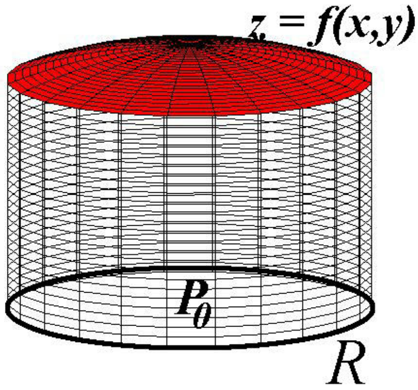
Conclusion: If f is a continuous function and $\int_a^b f(x)\ dx = 0$ for every interval $[a,\ b]$ then $f(x) = 0$ at all points x.



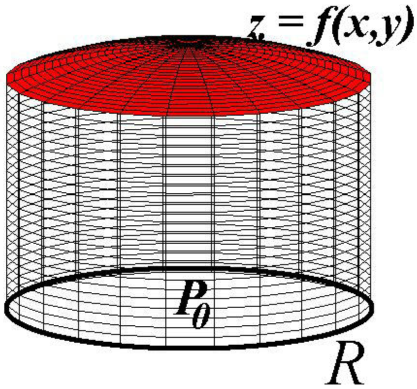
Suppose $z = f(x, y)$ is a continuous function and there was a point P_0 at which $f(P_0) > 0$. There would be a region R in the xy plane for which $f(x, y) > 0$



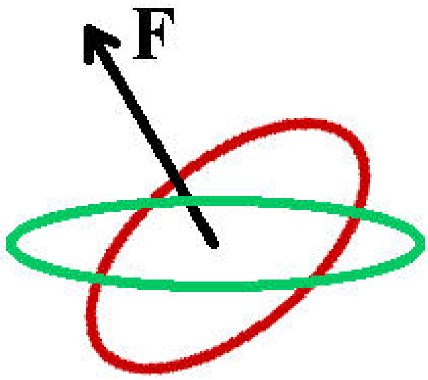
$$\int\int_R f(x,y)\, dA > 0$$



Conclusion:
If $f(x,y)$ is a continuous function and $\int\int_R f(x,y)\,dA = 0$ for *every* region R then $f(x,y) = 0$ at all points.

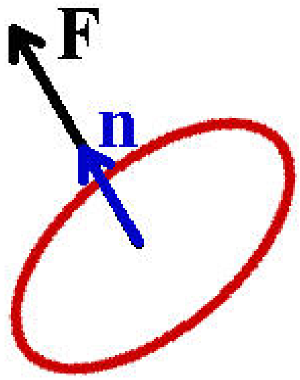


Suppose the surface integral of $\vec{\mathbf{F}}$ is 0 for any surface S regardless of size, shape or orientation of S.



$$\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS = 0$$

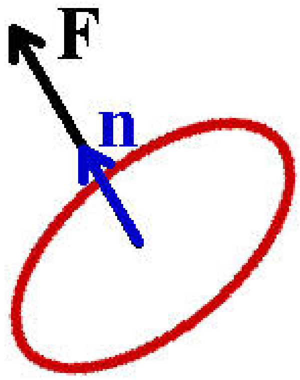
Pick S to be in a plane perpendicular to $\vec{\mathbf{F}}$ and orient it so that $\vec{\mathbf{n}}$ is in the same direction as $\vec{\mathbf{F}}$



$$\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS = 0$$

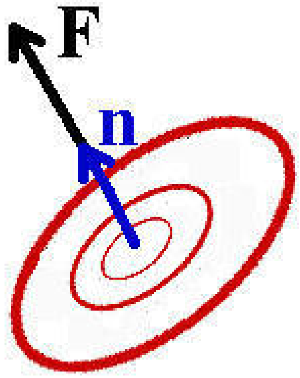
If $\vec{\mathbf{n}}$ and $\vec{\mathbf{F}}$ are in the same direction, then

$$\vec{\mathbf{F}} \bullet \vec{\mathbf{n}} = |\vec{\mathbf{F}}||\vec{\mathbf{n}}| \cos 0 = |\vec{\mathbf{F}}|(1)(1) = |\vec{\mathbf{F}}|$$



$$\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}} \, dS = \iint_S |\vec{\mathbf{F}}| \, dS = 0$$

$\iint_S |\vec{\mathbf{F}}|\, dS = 0$ no matter how small the loop.



This can only happen if $|\vec{\mathbf{F}}| = 0$

$$\sqrt{F_1^2 + F_2^2 + F_3^2} = 0$$

$$F_1 = 0 \qquad F_2 = 0 \qquad F_3 = 0$$

$$\vec{\mathbf{F}} = \langle 0,\ 0,\ 0 \rangle = \vec{\mathbf{0}}$$