Fourier Transform Formulas

Fourier Transforms

The Fourier transform of $f(x)$ and the inverse transform of $\hat{f}(\omega)$ are given by:

$$\mathcal{F}(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx \quad \mathcal{F}^{-1}(\hat{f}(\omega)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x}\, d\omega$$

The following Fourier Transforms are useful:

If $f(x) = \begin{cases} 1 & \text{for } -b < x < b \\ 0 & \text{otherwise} \end{cases}$ then $\mathcal{F}(f(x)) = \sqrt{\frac{2}{\pi}} \frac{\sin b\omega}{\omega}$

If $f(x) = \begin{cases} e^{-ax} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$ then $\mathcal{F}(f(x)) = \frac{1}{\sqrt{2\pi}(a + i\omega)}$

If $f(x) = e^{-|x|}$ then $\mathcal{F}(f(x)) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\omega^2}$

If $f(x) = \frac{1}{1+x^2}$ then $\mathcal{F}(f(x)) = \sqrt{\frac{\pi}{2}} e^{-|\omega|}$

If $f(x) = e^{-ax^2}$ then $\mathcal{F}(f(x)) = \frac{1}{\sqrt{2a}} e^{-\omega^2/(4a)}$

If $u = u(x,t)$ and $\hat{u}(\omega, t) = \mathcal{F}(u(x,t))$ then $\mathcal{F}\left(\frac{\partial u}{\partial t}\right) = \frac{\partial \hat{u}}{\partial t}$ and $\mathcal{F}\left(\frac{\partial u}{\partial x}\right) = i\omega \hat{u}(\omega, t)$