MA 441 Fall 2021 **Dr. E. Jacobs**

Practice Problems for Exam I

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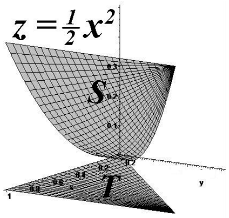
1. Let $\mathcal{T}$ be the triangular region in the xy plane with vertices $(0,\ 0,\ 0)$, $(1,\ 0,\ 0)$ and $(1,\ 1,\ 0)$. Let S be the portion of the surface $z = \frac{1}{2}x^2$ that lies directly above $\mathcal{D}$



Find the surface area of S. Show all work.

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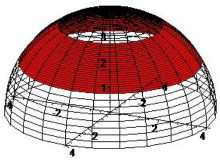
2. Let S be the portion of a sphere described by the following equation:

$$\langle x,\ y,\ z\rangle = \langle 4\cos\theta\sin\phi,\ 4\sin\theta\sin\phi,\ 4\cos\phi\rangle \quad \text{for } 0 \le \theta \le 2\pi \text{ and } \frac{\pi}{6} \le \phi \le \frac{\pi}{3}$$



a. Find the surface area of S

b. Suppose mass is distributed along S with a density function given by $\delta = \frac{1}{8}z$ kg/m^2. Find the total mass on S.

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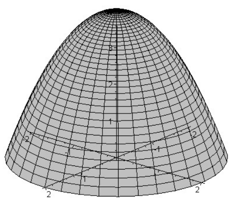
3. Let Ω be the portion of the parabolic surface $z = 1 - x^2 - y^2$ for $z \ge 0$. It can also be described by the parametric equation:

$$\vec{\mathbf{r}} = \left\langle u\cos\theta,\ u\sin\theta,\ 1 - u^2\right\rangle \quad \text{for } 0 \le u \le 1 \text{ and } 0 \le \theta \le 2\pi$$

Let B be the disk of radius 1 in the xy plane centered around the origin. Let S be the closed surface consisting of Ω and B.



Let $\vec{\mathbf{F}} = \langle -y,\ x,\ z+1\rangle$.

Calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ by adding $\iint_\Omega \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ to $\iint_B \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$.

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4. Let $\vec{\mathbf{F}}$ and surface S be defined exactly as in problem 3. Use the Divergence Theorem to calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$

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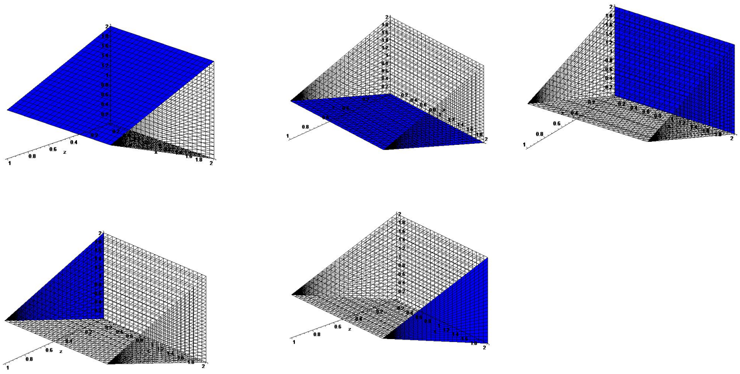
5. Let V be the three dimensional region below the plane $z = 2 - x$ and above the plane $z = x$ for $0 \leq x \leq 1$ and $0 \leq y \leq 2$. Let S be the *closed surface* surrounding V. There are five surfaces that make up S.



Let $\vec{\mathbf{F}} = \langle x,\ 0,\ z \rangle$. Calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ by adding up the surface integrals for all five surfaces. *Hint: You will save some time if you can see which of these five surface integrals will equal 0.*

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6. Let $\vec{\mathbf{F}}$ and S be defined exactly as in problem 5. Calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ using the Divergence Theorem.

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7. Let $\phi(x, y) = xy - \frac{x^2}{2}$ and $\vec{\mathbf{F}} = \left(x - y^2\right)\vec{\mathbf{j}} + \left(z - x^2\right)\vec{\mathbf{k}}$. Calculate each of the following:

a) $\operatorname{div} \vec{\mathbf{F}}$ b) $\operatorname{curl} \vec{\mathbf{F}}$ c) $\nabla\phi$ d) $\nabla^2\phi$
