MA 441 Spring 2022 Practice Problems for Exam III

1. Find all functions $X(x)$ and $T(t)$ so that $u(x,t) = X(t)T(t)$ is a *non-trivial* solution of the partial differential equation:

$$\frac{\partial u}{\partial t} + 2u = \frac{\partial^2 u}{\partial x^2}$$

as well as the boundary conditions: $u(0,t) = 0$ and $u\left(\frac{\pi}{2},\ t\right) = 0$.

2. Use Euler's formula to simplify the following expression:

$$e^{x+2iy} + e^{x-2iy}$$

3. Classify each of the following functions as being *even*, *odd* or *neither even nor odd*:

a) $\sin x \sin 2x$ **b)** $x^3 + x$ **c)** $x|x|$ **d)** $x + |x|$ **e)** $\sin x(1 + \cos x)$

4. Find the Fourier series for the following function for $-1 \le x \le 1$:

$$f(x) = \begin{cases} 1 + x & \text{for } \quad 0 \le x \le 1 \\ -1 + x & \text{for } -1 \le x < 0 \end{cases}$$

5. Find the Fourier series for the4 function $f(x) = \pi - |x|$ on the interval $-\pi \le x \le \pi$

6. Find the solution $u = u(x,t)$ of the following partial differential equation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

where $0 \le x \le 1$ and u satisfies each of the following boundary conditions:

$$u(0,t) = 0 \qquad\qquad u(1,t) = 0$$

and initial conditions:

$$u(x,0) = 0 \qquad\qquad \tfrac{\partial u}{\partial t}(x,0) = \sin 2\pi x + 2\sin 4\pi x$$

7. Let $u = u(x, y)$ be the solution of Laplace's equation, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 \leq x,\ y \leq \pi$. It can be shown that if u satisfies the boundary conditions $u(0,\ y) = 0$, $u(\pi,\ y) = 0$ and $u(x,\ 0) = 0$, then $u(x, y)$ will be given by the following Fourier expression:

$$u(x, y) = \sum_{n=1}^{\infty} b_n \left(e^{ny} - e^{-ny}\right) \sin nx$$

Determine the formula for the coefficient b_n if u also must satisfy the boundary condition $\frac{\partial u}{\partial y}(x,\ 0) = 1$ for all x in the interval $0 \leq x \leq \pi$.