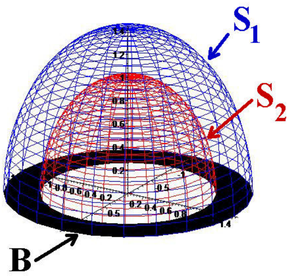
Practice Problems for Final Exam

1. Let V be the region between the two spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=2$ for $z \geq 0$. Let S be the closed surface surrounding V. S consists of three surfaces, S_1, S_2 and B, shown below.



Let $\vec{\mathbf{F}} = \langle 0,\ 0,\ 2z^2 \rangle$. Calculate the surface integral $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ by adding up the surface integrals over S_1, S_2 and B.

---

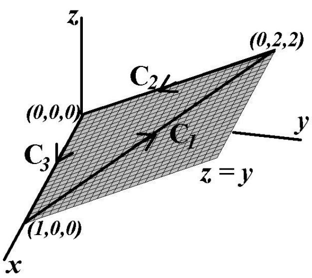
2. Let $\vec{\mathbf{F}}$ and S be defined exactly as in problem 1. Calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ by using the Divergence Theorem.

---

3. Let $\vec{\mathbf{F}} = \langle 2xz+z,\ 0,\ x^2+x \rangle$. Is $\vec{\mathbf{F}}$ a conservative vector field? If it is, find a scalar valued function ϕ so that $\vec{\mathbf{F}} = \nabla\phi$. If $\vec{\mathbf{F}}$ is not a conservative vector field, prove it.

---

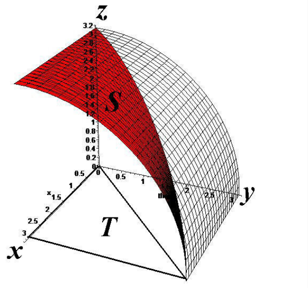
4. Let $\vec{\mathbf{F}} = \langle -y,\ x,\ z^2 \rangle$.
Let C_1 be the straight line path from $(1,\ 0,\ 0)$ to $(0,\ 2,\ 2)$
Let C_2 be the straight line path from $(0,\ 2,\ 2)$ to $(0,\ 0,\ 0)$
Let C_3 be the straight line path from $(0,\ 0,\ 0)$ to $(1,\ 0,\ 0)$
Together, these paths form a closed triangular loop C. All points on C lie on the plane $z=y$.



a) Calculate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ by adding up the line integrals over C_1, C_2 and C_3.

b) Calculate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ using Stokes' Theorem.

5. Let T be the triangular region in the xy plane with vertices $(0,\ 0,\ 0)$, $(\pi,\ 0,\ 0)$ and $(\pi,\ \pi,\ 0)$. Let S be the portion of the cylinder $x^2 + z^2 = \pi^2$ that lies directly above T. Calculate the surface area of S



---

6. Which of the following functions would be given as a Fourier sine series?

a) $f(x) = \sin x \cos x \tan^2 x$ **b)** $f(x) = 3x$ **c)** $f(x) = x^2 - x^3$
d) $f(x) = \sin x + \cos x$

---

7. Find the Fourier series for the following function $f(x)$ on the interval $-1 \leq x \leq 1$.

$$f(x) = x + \sin \pi x$$

---

8. Let $u(x, t)$ be the solution of $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0$, where $u(x, t)$ satisfies the boundary conditions $u(0,\ t) = 0$ and $u(\pi,\ t) = 0$. Obtain the solution in the form of a Fourier series by first looking for solutions of the form $X(x)T(t)$. There are no initial conditions given, so your Fourier series answer will have coefficients that remain undetermined.