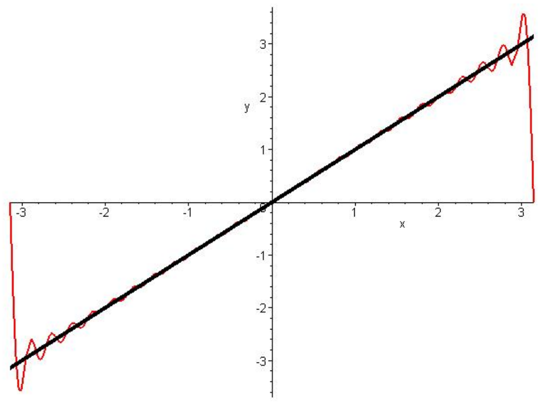
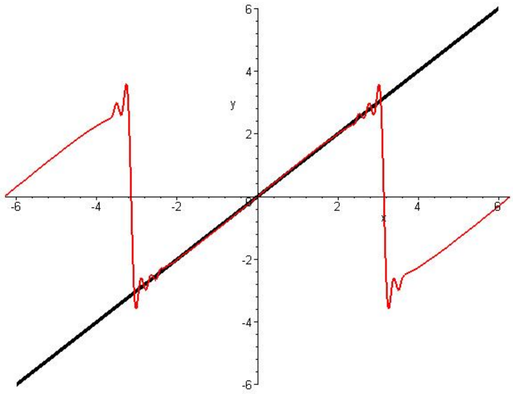
$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right)$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

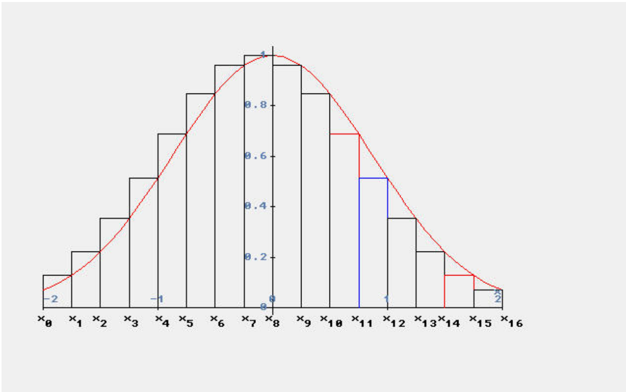


y
x
3
2
1
-1
-2
-3

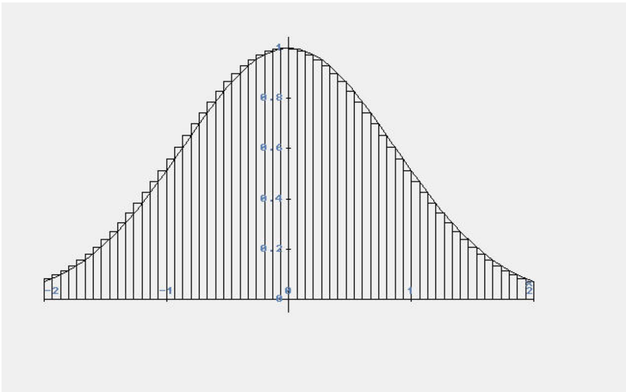


6
4
y
2
-6
-4
-2
0
2
4
6
x
-2
-4
-6

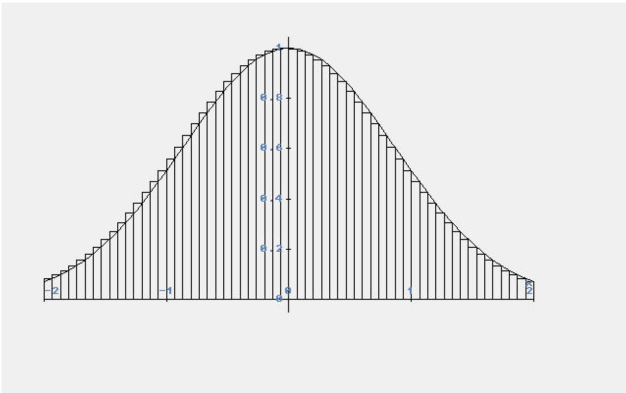
$$\sum_{n=1}^{16} f(x)\,\Delta x$$



$$\int_a^b f(x)\,dx = \lim_{N\to\infty}\sum_{n=1}^{N} f(x)\,\Delta x$$



$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{n=1}^{N} f(x)\,\Delta x$$



$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

Let $\omega_n = \frac{n\pi}{L}$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos \omega_n x + b_n \sin \omega_n x)$$

Let $\omega_n = \frac{n\pi}{L}$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \omega_n x \, dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \omega_n x \, dx$$

1. $$e^{i\theta} = \cos\theta + i\sin\theta$$

2. $$e^{-i\theta} = \cos\theta - i\sin\theta$$

$(1) + (2)$ $$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

$(1) - (2)$ $$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) = -\frac{i}{2}\left(e^{i\theta} - e^{-i\theta}\right)$$

$$\int_0^1 x^2 \, dx = \frac{1}{3}$$

$$\int_0^1 y^2 \, dy = \frac{1}{3}$$

$$\int_0^1 v^2 \, dv = \frac{1}{3}$$

$$\int_0^1 \omega^2 \, d\omega = \frac{1}{3}$$

The following integrals all have the same value:

$$\int_{-\infty}^{0} \hat{f}(v) e^{ivx}\, dv$$

$$\int_{-\infty}^{0} \hat{f}(\omega) e^{i\omega x}\, d\omega$$

$$
\begin{aligned}
f(x) &= \int_0^{\infty} \hat{f}(\omega)e^{i\omega x}\,d\omega + \int_{-\infty}^{0} \hat{f}(v)e^{ivx}\,dv \\
&= \int_0^{\infty} \hat{f}(\omega)e^{i\omega x}\,d\omega + \int_{-\infty}^{0} \hat{f}(\omega)e^{i\omega x}\,d\omega \\
&= \int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x}\,d\omega
\end{aligned}
$$

Fourier Transform Pairs

$$\hat{f}(\omega) = \mathcal{F}(f(x)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx$$

$$f(x) = \mathcal{F}^{-1}(\hat{f}(\omega)) = \int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x}\,d\omega$$

Fourier Transform Pairs - Alternate

$$\hat{f}(\omega) = \mathcal{F}(f(x)) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx$$

$$f(x) = \mathcal{F}^{-1}(\hat{f}(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x}\,d\omega$$

Fourier Transform Pairs - Alternate

$$\hat{f}(\omega) = \mathcal{F}(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$$

$$f(x) = \mathcal{F}^{-1}(\hat{f}(\omega)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x}\, d\omega$$

Fourier Transform Pairs - Zill and Wright

$$\hat{f}(\alpha) = \mathcal{F}(f(x)) = \int_{-\infty}^{\infty} f(x)e^{i\alpha x}\, dx$$

$$f(x) = \mathcal{F}^{-1}(\hat{f}(\alpha)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\alpha)e^{-i\alpha x}\, d\alpha$$