Advanced Engineering Mathementics I

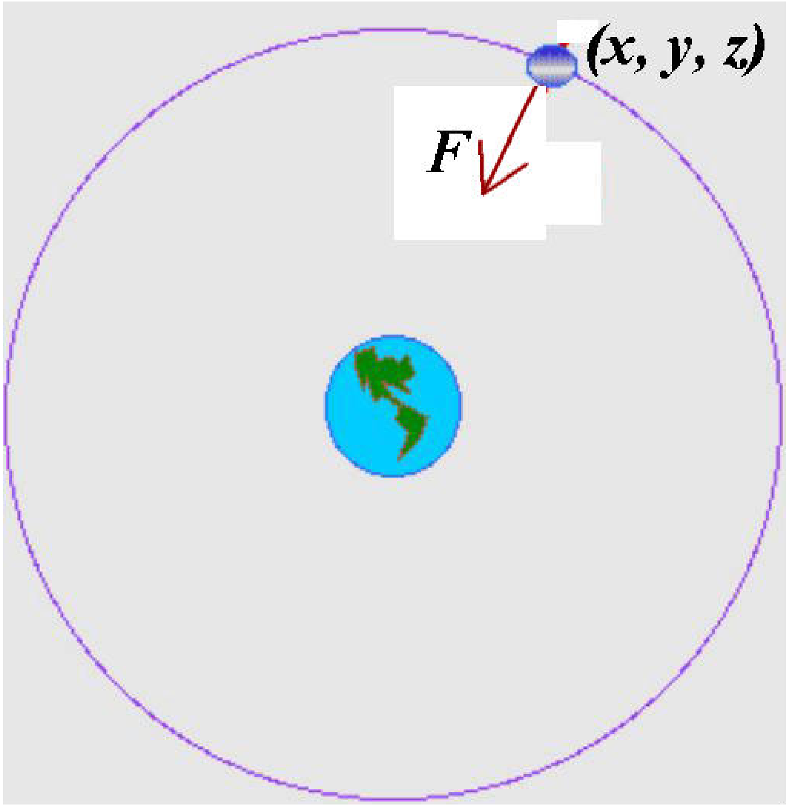
Prerequisites:
MA 241, MA 242, MA 243

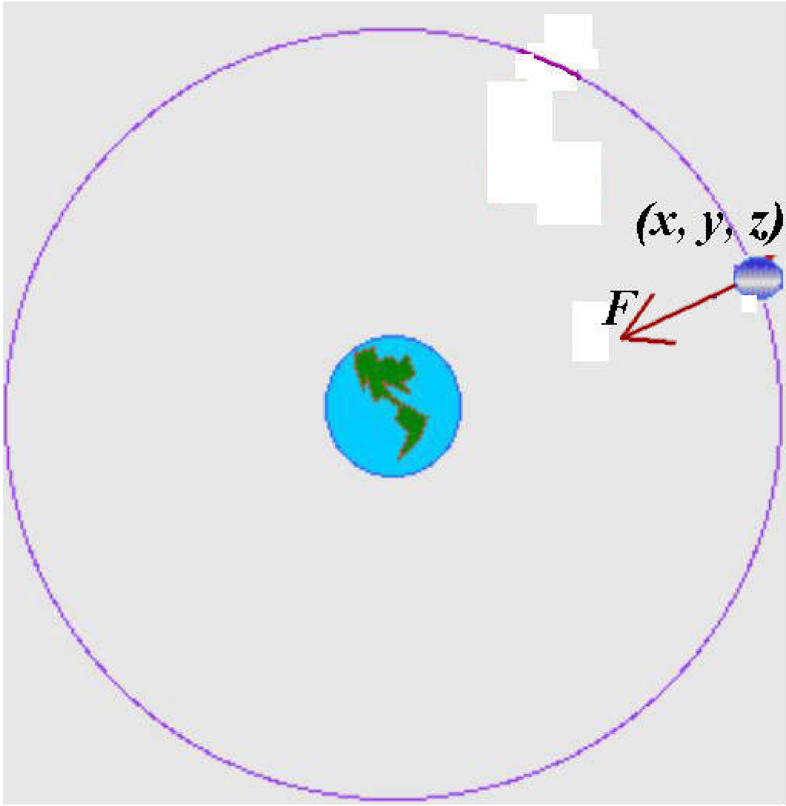
MA 345 is helpful but not required.
You will need a strong background in MA 243

MA 441 covers the following topics:

1. **Vector Calculus**

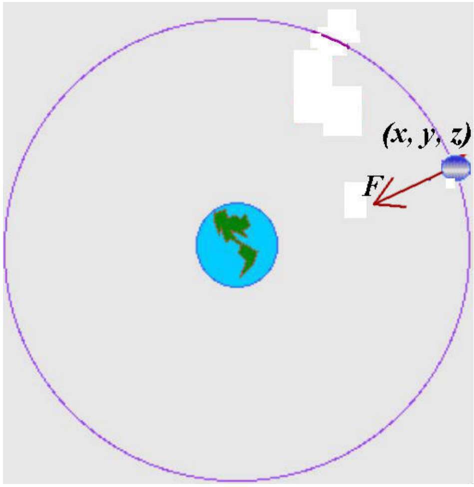
2. **Fourier Series**


(x, y, z)
F


(x, y, z)
F

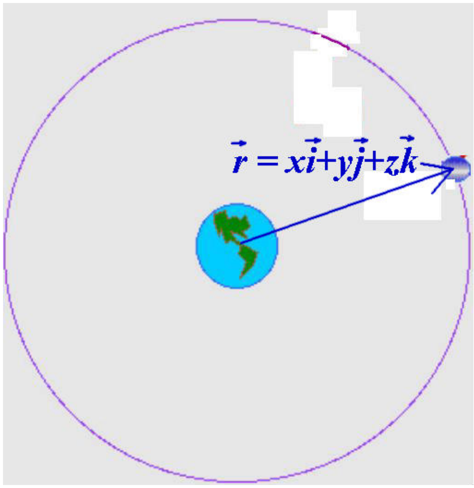
The force $\vec{\mathbf{F}}$ is a function of location $(x,\ y,\ z)$

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}(x, y, z)$$

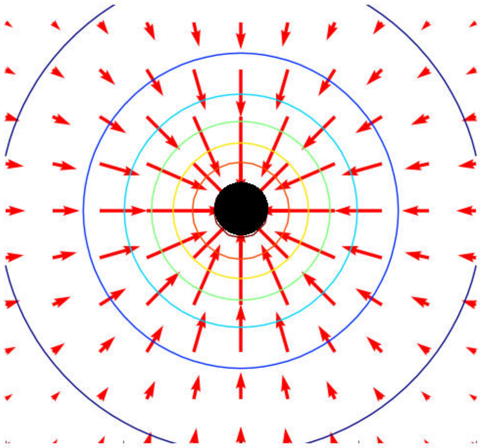


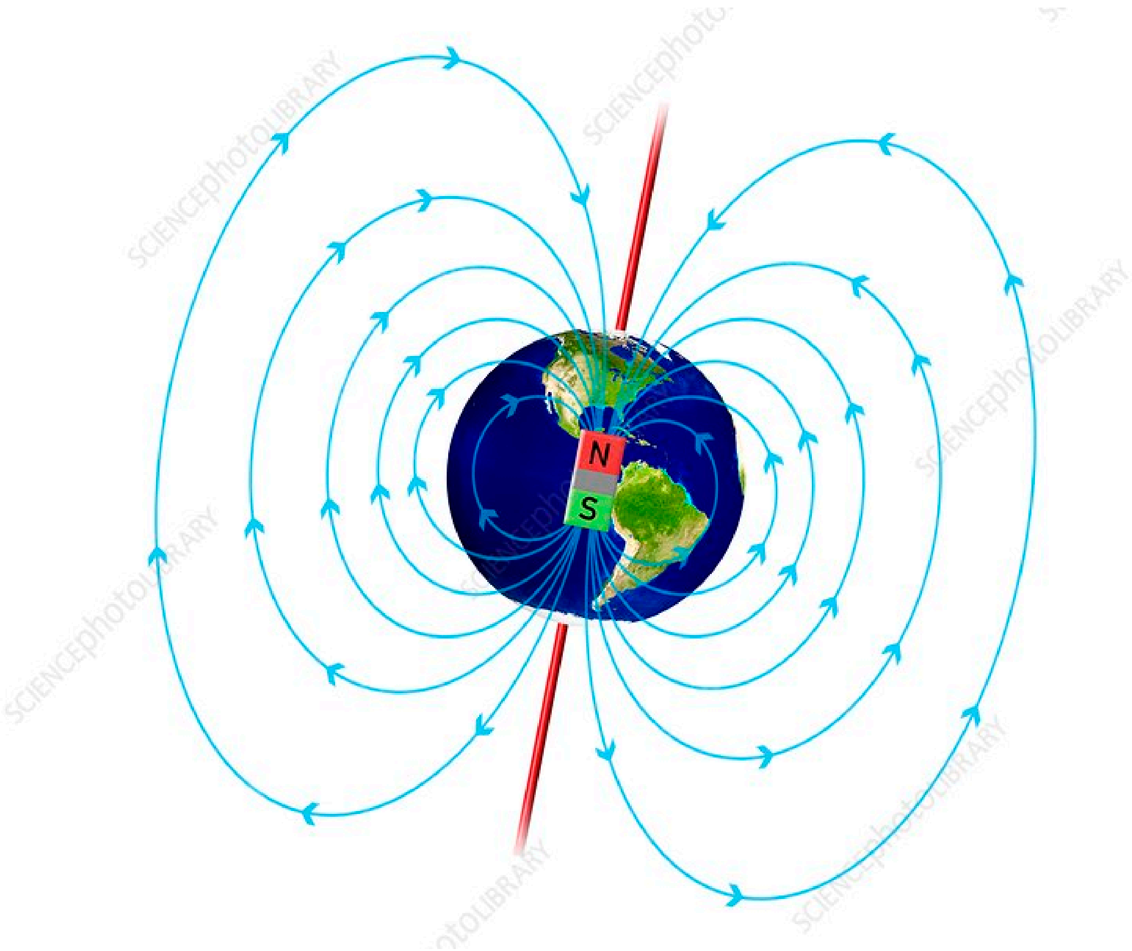
Let $\vec{\mathbf{r}}$ be the location vector

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}(\vec{\mathbf{r}})$$



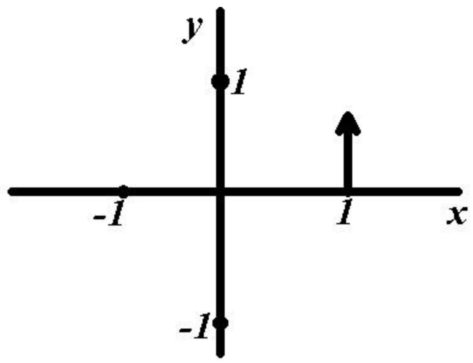
A vector-valued function of a vector variable is called a *vector field*




N
S

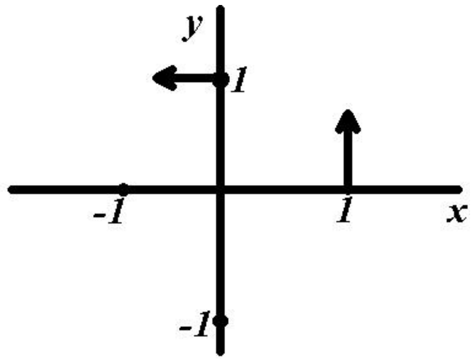
$$\text{Let } \vec{\mathbf{F}} = -y\vec{\mathbf{i}} + x\vec{\mathbf{j}}$$

Plot at (1, 0)



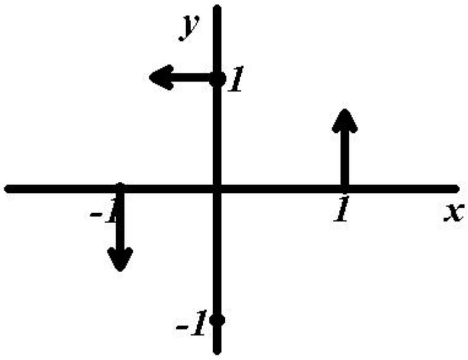
$$\text{Let } \vec{\mathbf{F}} = -y\vec{\mathbf{i}} + x\vec{\mathbf{j}}$$

Plot at (0, 1)

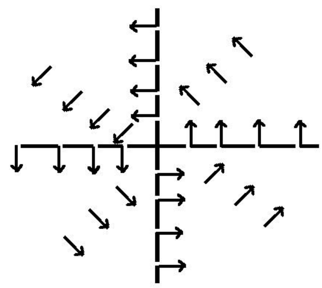


$$\text{Let } \vec{\mathbf{F}} = -y\vec{\mathbf{i}} + x\vec{\mathbf{j}}$$

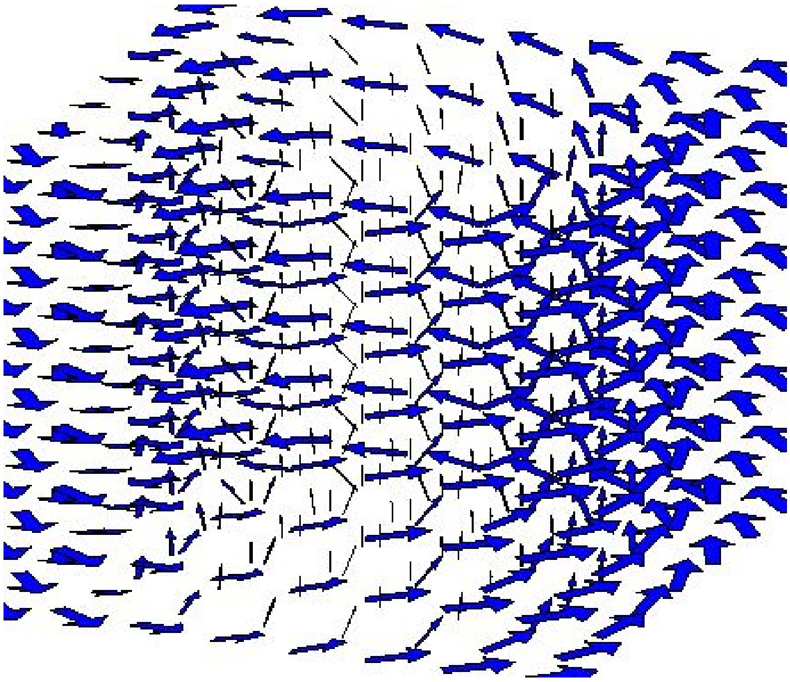
Plot at $(-1,\ 0)$



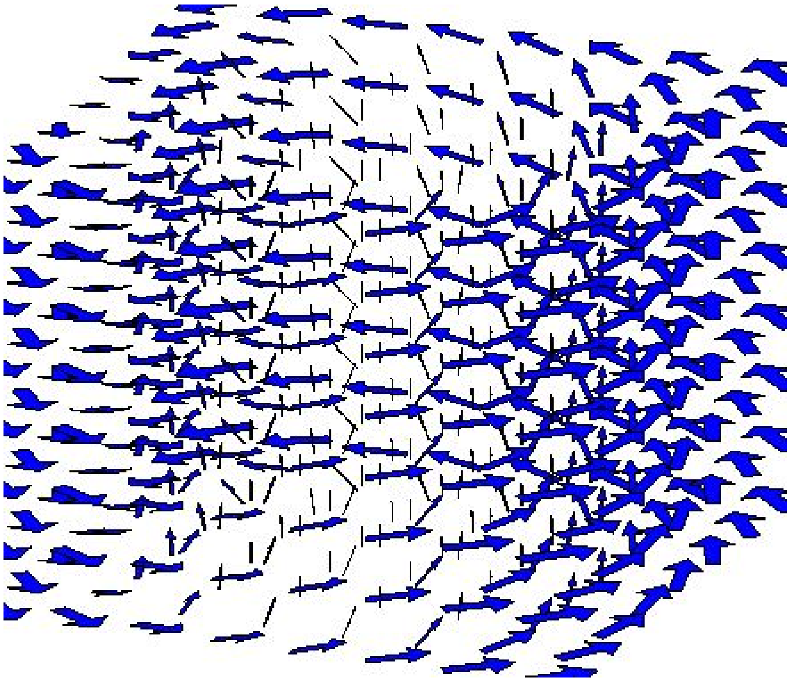
$$\text{Let } \vec{\mathbf{F}} = -y\vec{\mathbf{i}} + x\vec{\mathbf{j}}$$



Let $\vec{\mathbf{F}} = -y\vec{\mathbf{i}} + x\vec{\mathbf{j}} + 1\vec{\mathbf{k}}$



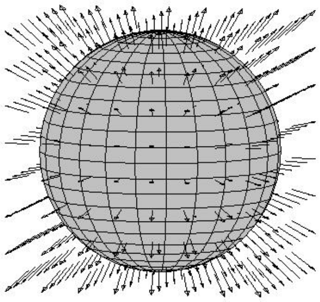
Let $\vec{\mathbf{F}} = \langle -y,\ x,\ 1 \rangle$

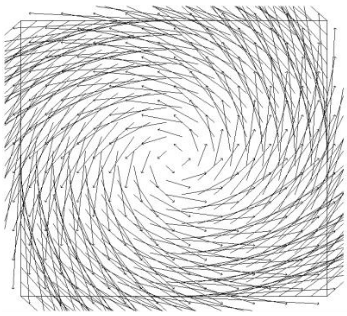


Vector Calculus

Derivatives:

$\mathbf{div}\,\vec{\mathbf{F}}$ $\qquad\qquad$ $\mathbf{curl}\,\vec{\mathbf{F}}$





Vector Calculus

Integrals:

Surface Integrals

$$\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$$

Line Integrals

$$\int_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$$

Generalizations of the Fundamental Theorem

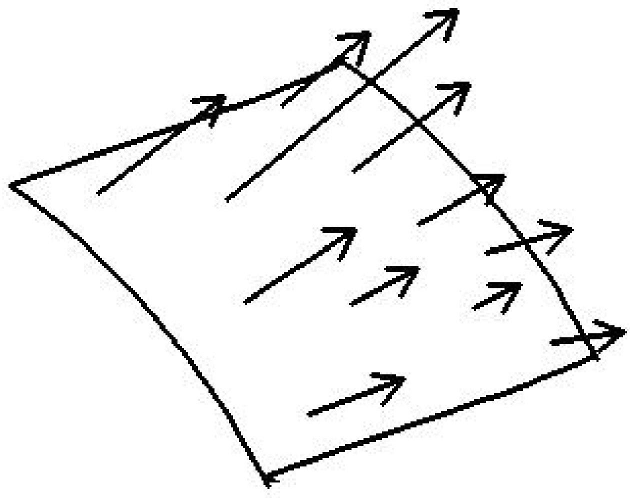
The Divergence Theorem

$$\iiint_V \mathbf{div}\, \vec{\mathbf{F}}\, dV = \iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$$

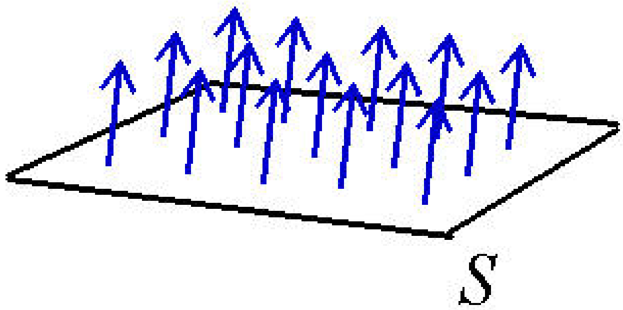
Stokes' Theorem

$$\iint_S \mathbf{curl}\, \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$$

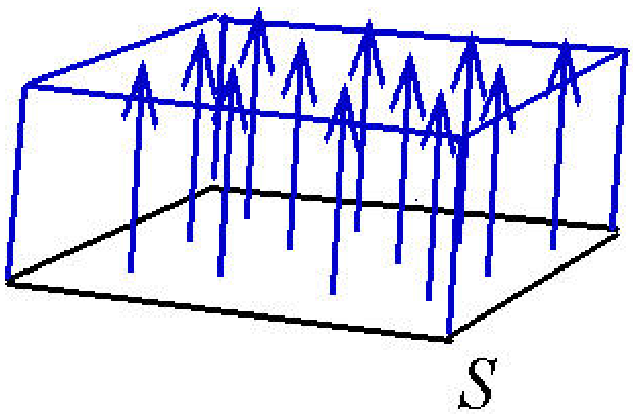
Vector field passing through a surface



Let $\vec{\mathbf{v}}$ be the velocity of a fluid (in m/sec). What volume of fluid moves through the surface in time Δt



Let $\vec{\mathbf{v}}$ be the velocity of a fluid (in m/sec). The volume of fluid that passes through a surface S in Δt seconds is $|\vec{\mathbf{v}}|\Delta t \cdot (\text{Area of } S)$



Let $\vec{\mathbf{v}}$ be the velocity of a fluid (in m/sec). The volume of fluid that passes through a surface S in Δt seconds is $|\vec{\mathbf{v}}|\Delta t \cdot$ (Area of S)