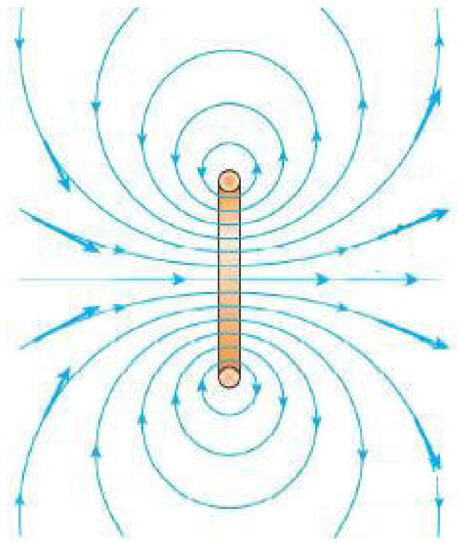
Stokes' Theorem Applications
Electric and Magnetic Fields

Dr. Elliott Jacobs



Integral Form:

$$\iint_S \vec{\mathbf{B}} \bullet \vec{\mathbf{n}} \, dS = 0$$

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}} \, dS = \frac{1}{\epsilon_0} q$$

Integral Form:

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$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = \frac{1}{\epsilon_0} q$$

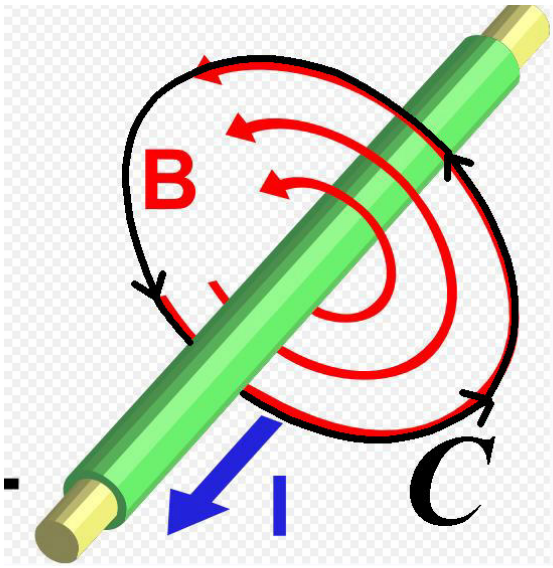
Differential Form:

$$\nabla \bullet \vec{\mathbf{B}} = 0$$

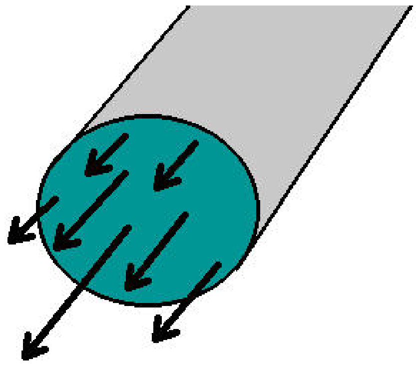
$$\nabla \bullet \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho$$

Ampere's Law

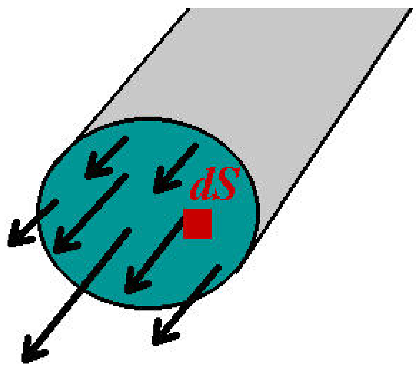
$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{r}} = \mu_0 I$$



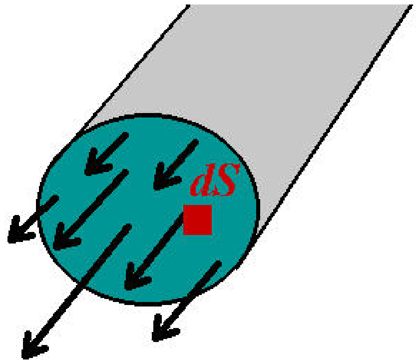
Let $\vec{\mathbf{J}}$ be the current density vector



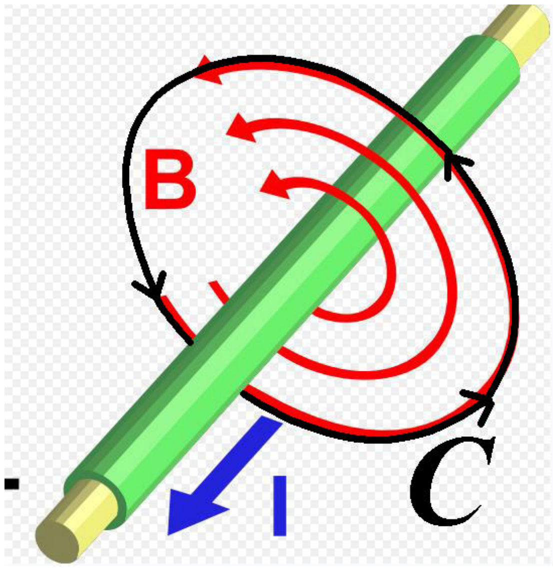
$\vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$ is the amount of current (in amperes) that passes through one small section of area dS



$$I = \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$



$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{r}} = \mu_0 I = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$



$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{r}} = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$

$$\iint_S (\nabla \times \vec{\mathbf{B}}) \bullet \vec{\mathbf{n}}\, dS = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$

$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{r}} = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$

$$\iint_S (\nabla \times \vec{\mathbf{B}}) \bullet \vec{\mathbf{n}}\, dS = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$

$$\iint_S (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{J}}) \bullet \vec{\mathbf{n}}\, dS$$

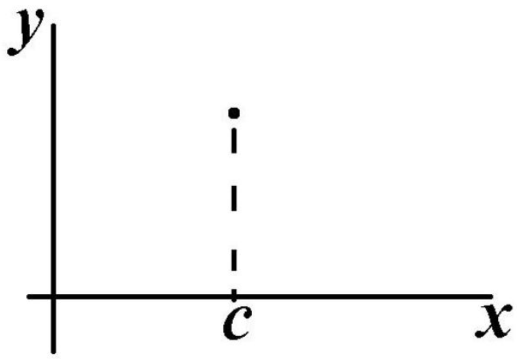
$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{r}} = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$

$$\iint_S (\nabla \times \vec{\mathbf{B}}) \bullet \vec{\mathbf{n}}\, dS = \mu_0 \iint_S \vec{\mathbf{J}} \bullet \vec{\mathbf{n}}\, dS$$

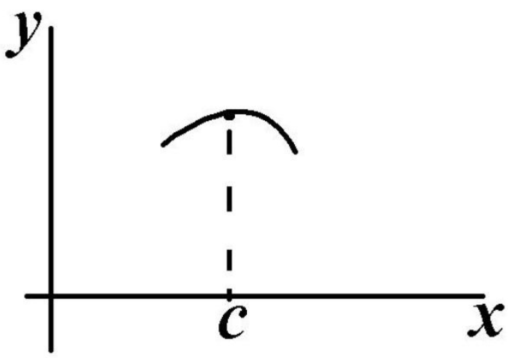
$$\iint_S (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{J}}) \bullet \vec{\mathbf{n}}\, dS$$

$$\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{J}} = \vec{\mathbf{0}}$$

Suppose there were one point $x = c$ where $f(c) \neq 0$



If f is a continuous function and $f(x) \neq 0$ at some point then $\int_a^b f(x)\,dx \neq 0$ for some interval around that point. Equivalently, if $\int_a^b f(x)\,dx = 0$ for every interval, then $f(x) = 0$ for all x



$$\iint_S (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{J}}) \bullet \vec{\mathbf{n}}\, dS = 0$$

If this is true for *every* surface S and $\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{J}}$ varies continuously then

$$\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{J}} = \vec{\mathbf{0}} \quad \text{at all points}$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}}$$

Ampere's Law

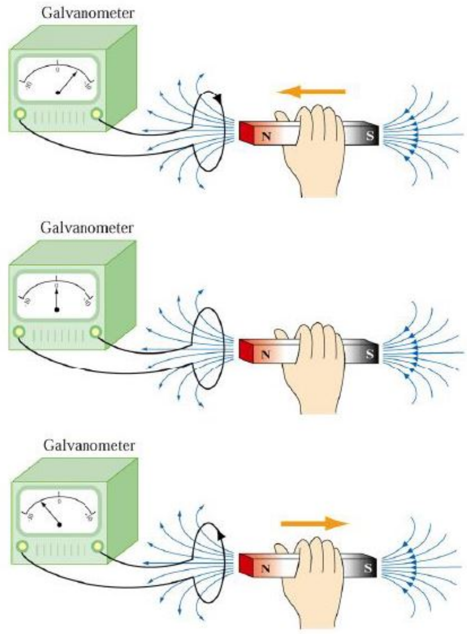
Integral form:

$$\oint_C \vec{\mathbf{B}} \bullet d\vec{\mathbf{r}} = \mu_0 I$$

Differential form:

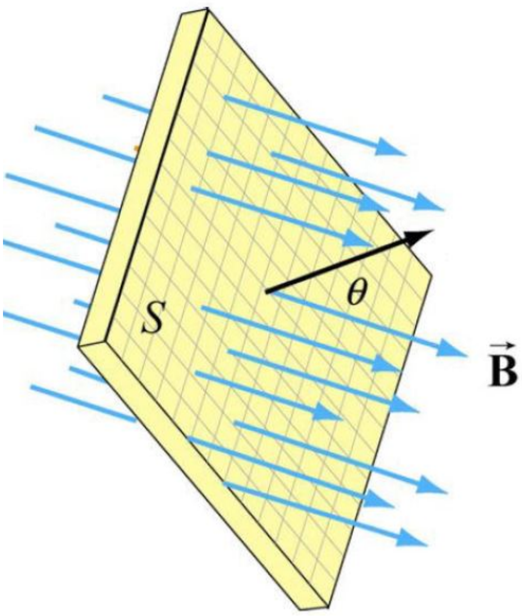
$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}}$$

Electromagnetic Induction



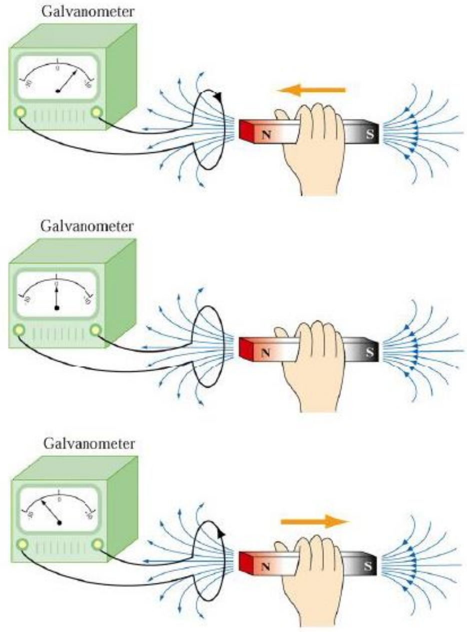
Magnetic Flux Through Surface S

$$\Phi_S = \iint_S \vec{\mathbf{B}} \bullet \vec{\mathbf{n}} \, dS$$



Faraday's Law

$$-\frac{d\Phi_S}{dt} = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$



$$-\frac{d\Phi_S}{dt} = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\frac{d}{dt} \iint_S \vec{\mathbf{B}} \bullet \vec{\mathbf{n}} \, dS = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

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$$-\iint_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet \vec{\mathbf{n}} \, dS = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$G(t) = \int_a^b f(x, t)\, dx$$

$$G(t+h) = \int_a^b f(x, t+h)\, dx$$

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$$G(t+h) - G(t) = \int_a^b (f(x, t+h) - f(x, t))\, dx$$

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$$G(t) = \int_a^b f(x, t)\, dx$$

$$\frac{G(t+h) - G(t)}{h} = \int_a^b \frac{f(x, t+h) - f(x, t)}{h}\, dx$$

Take the limit as $h \to 0$

$$G(t+h) = \int_a^b f(x, t+h)\, dx$$

$$G(t) = \int_a^b f(x, t)\, dx$$

$$\frac{G(t+h) - G(t)}{h} = \int_a^b \frac{f(x, t+h) - f(x, t)}{h}\, dx$$

$$G'(t) = \int_a^b \frac{\partial f}{\partial t}\, dx$$

$$-\frac{d\Phi_S}{dt} = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\frac{d}{dt} \iint_S \vec{\mathbf{B}} \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\iint_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\iint_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet \vec{\mathbf{n}}\, dS = \iint_S (\nabla \times \vec{\mathbf{E}}) \bullet \vec{\mathbf{n}}\, dS$$

$$-\frac{d\Phi_S}{dt} = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\frac{d}{dt} \iint_S \vec{\mathbf{B}} \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\iint_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet \vec{\mathbf{n}}\, dS = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

$$-\iint_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet \vec{\mathbf{n}}\, dS = \iint_S (\nabla \times \vec{\mathbf{E}}) \bullet \vec{\mathbf{n}}\, dS$$

$$0 = \iint_S \left(\frac{\partial \vec{\mathbf{B}}}{\partial t} + \nabla \times \vec{\mathbf{E}} \right) \bullet \vec{\mathbf{n}}\, dS$$

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$$\vec{\mathbf{0}} = \frac{\partial \vec{\mathbf{B}}}{\partial t} + \nabla \times \vec{\mathbf{E}}$$

Faraday's Law

Integral form:

$$-\frac{d\Phi_S}{dt} = \oint_C \vec{\mathbf{E}} \bullet d\vec{\mathbf{r}}$$

Differential form:

$$-\frac{\partial \vec{\mathbf{B}}}{\partial t} = \nabla \times \vec{\mathbf{E}}$$